Technical report:

**Stochastic Single and Multizone Models of a Hybrid Ventilated Building – A Monte Carlo Simulation Approach**

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1 Introduction

In the design of building ventilation, prediction of system performance with regard to indoor air quality, thermal comfort and energy consumption is traditionally based on deterministic models and loads. A deterministic approach implies that all input parameters and model coefficients are 100% certain with zero spread. In practice this is not the case, for instance inhabitant behaviour and internal loads may vary significantly and external loads such as wind and external temperature are obviously stochastic in nature. In a conventional deterministic analysis, one would use a single representative value of each design parameter or input load, or use a relatively short time series, e.g. a design reference year, to examine the behaviour of the ventilation system. However, by doing so, the influence of the stochastic load processes, e.g. the external weather climate will not be assigned properly. A stochastic analysis will provide in-depth information about the variability and the mutual dependence between the output parameters, thus enabling one to a realistic interpretation of the results.
This report deals with probabilistic single and multizone air flow modelling to predict the random nature of air flow and thus the indoor air quality in buildings. For probabilistic thermal building simulation, see (Brohus et al., 2002b).

The air flow through a building is driven by natural driving forces from wind pressure and air density differences (stack effect) and mechanical driving forces (electrically powered fan). The random natures of the loads are especially important in the case of naturally and hybrid ventilated buildings. Here, the external weather climate more directly affects the indoor air quality depending on the geometry of the building, building location and orientation, internal partitions, location and size of openings, etc.

An crucial point when considering hybrid ventilated buildings is the control strategy, as some kind of control is needed to keep the indoor climatic parameters within acceptable bounds switching between natural and mechanical modes (driving forces). The control is a kind of feedback loop to the model representing the hybrid ventilated enclosure and a stochastic model without inclusion of control strategies is of limited use for practical applications.

As a starting point, a single zone model is considered, where the building is assumed to consists of a single zone and one air flow path, see Chapter 3. Two control mechanisms, a damper and a fan, are included in order to control the air flow towards a requested constant value. The Monte Carlo Simulation (MCS) approach, as described in Chapter 2, is used to generate multiple realisations of air flow and control parameter time series, based on similar simulated time series of the load parameters. For probabilistic load modelling, see (Brohus et al., 2002a). The statistics of the air flow and control parameters, i.e. mean values, standard deviations and distribution functions, can then be obtained by statistical sampling among the simulated time series.

The single zone air flow model is capable of predicting the air flow through an enclosure, if it is assumed to consist of one well-mixed compartment. A simple application for this type of model is the single story, family house with no internal partitions, corresponding to the situation where all internal doors are opened. Another application is an atrium or other large enclosures.

Most buildings, however, cannot be characterised as a single zone. If multiple air flow paths can be recognised within the building, a multizone model is more appropriate taking into account the internal partitions. Such a model can be regarded as a generalisation of a single zone model. Instead of obtaining one nonlinear air flow equation for a single zone, a nonlinear equation system is formulated in the multizone case. Thus, a nonlinear air flow equation is formulated for each zone and the equations are solved for the unknown zone pressures after which the air flows between the zones can be calculated, (Herrlin and Allard, 1992). A review of several multizone models is given by Feustel and Dieris (1992).

A multizone model is typically formulated in general terms, i.e. an arbitrary number of zones and partitions are possible, and the air flow equations are assembled and solved using a computer code, see e.g. Feustel (1999). The air flow paths are represented by equations modelling the air flows as functions of the pressure differences over the openings. Typically, a library of semi-empirical relations are provided to model resistance due to internal and external openings, duct systems, stairwells, etc. Mechanical devices like fans are modelled to represent pressure increases between the internal zones, or between the interior and the external environment.

In Chapter 4, a multizone model is presented using a single type of equation to model the air flow paths. The model is formulated in general terms, enabling the inclusion of other types of components and control strategies in future work. The MCS approach is also used on the multizone
model to estimate the statistics of the air flows, i.e. mean values, standard deviations and distribution functions.

Both the single zone and the multizone model are demonstrated by test cases.

2 Monte Carlo Simulation approach

In the Monte Carlo Simulation (MCS) approach, multiple input time series are generated according to the statistics of the input parameters (Gentle 1998; Ogorodnikov and Prigarin, 1996). Based on each input time series, a corresponding time series of the output variables, e.g. air flow, can be obtained by the building model, in this case a single zone or multizone model. Based on multiple realisations output statistics are obtained, i.e. mean values, standard deviations and distribution functions. In this Chapter, it is first shown how the input can be simulated, taking into account the distribution functions of the various input parameters, their mutual dependence and the dependence in time (cross/auto-correlation). Then it is shown how the statistics of the output can be obtained by statistical sampling. The notation in this chapter is kept general in order to show the general applicability of the approach. In practice, an MCS module can be applied as a black box on a given method, in this case a single or multizone air flow model.

2.1 Simulation of stochastic vector processes

A \( n \)-dimensional stochastic vector process, \( \mathbf{X}(t) \), is simulated \( N \) times by the Monte Carlo Simulation approach. In order to simulate the stochastic vector process using discrete random variables, the process is discretised in time. Thus, each time series is represented by \( M \) discrete points in time, \( \mathbf{x}_r(t_k), r = 1, 2, \ldots, N, k = 1, 2, \ldots, M \), using a statistical description of the input. This is achieved using the marginal distribution functions, \( F_{X_i}(t_k), i = 1, \ldots, n \), defining the statistical properties for each instance in time for each component of the vector process. Furthermore, it is assumed that the cross-correlation coefficient function matrix, \( \mathbf{\rho}_{\mathbf{X} \mathbf{X}} \), is a function solely of the time lag, \( \tau = t_2 - t_1 \), i.e. \( \mathbf{\rho}_{\mathbf{X} \mathbf{X}} = \mathbf{\rho}_{\mathbf{X} \mathbf{X}}(\tau) \). The cross-correlation coefficient function matrix defines the mutual dependence between the components and in time.

Figure 2.1 shows a realisation of two stochastic processes, \( X(t) \) and \( Y(t) \). If temporal correlation (dependence) exist, the correlation between values of the processes at times \( t_1 \) and \( t_2 \) is expected to be larger than corresponding values at times \( t_1 \) and \( t_3 \), as the distance in time is larger in the latter case.
Figure 2.1 Principle sketch showing a realisation of two stochastic processes $X(t)$ and $Y(t)$.

In Figure 2.2 is sketched the cross and auto-correlation coefficient functions corresponding to the processes $X(t)$ and $Y(t)$.

Figure 2.2 Principle sketch showing cross and auto correlation coefficient functions for processes $X(t)$ and $Y(t)$. 

\[ \rho_{XX}(\tau) \]
\[ \rho_{XY}(\tau) \]
\[ \rho_{YY}(\tau) \]
The auto-correlation coefficient functions (the upper and lower curves) decrease from a value of one corresponding to zero time lag and thus full correlation. For infinite time lag, the functions become zero, corresponding to the fact that the processes will not be influenced of what have happened for infinite time ago. The cross-correlation coefficient function (centre curve) shows similar behaviour, but decreases from the value corresponding to the correlation between the two processes at zero time lag.

Application of multiple input time series on a deterministic building model, generates corresponding output time series, \( y_r(t_k) , r = 1, 2, \ldots, N, k = 1, 2, \ldots, M \), of a \( m \)-dimensional stochastic vector process, \( Y(t) \). The deterministic model can be either analytical or numerical (in the present case a single or multizone model). Finally, a statistical description of the output quantities in form of marginal output distribution functions, \( F_i(t), i = 1, \ldots, m \), and the auto/cross-correlation coefficient function matrix, \( \rho_{YY}(\tau) \), can be obtained by statistical sampling among the simulated time series.

The components of the input vector could be loads necessary to perform an air flow simulation, e.g. external air temperature, wind speed, etc., whereas the output are the corresponding air flows, pressure distributions, energy consumption, etc.

In the present work, the so-called Nataf transformation, see Kiureghian and Liu (1986), is used to establish a mapping between a realisation of the discretised \( n \)-dimensional vector process, \( x(t_k) , k = 1, 2, \ldots, M \), and a corresponding realisation of a \( n \)-dimensional discretised standard normal vector process, \( u(t_k) , k = 1, 2, \ldots, M \), with independent components. The purpose of the mapping is that it is a relatively simple task to simulate normal distributed random numbers on the computer. Such numbers are generated by a random number generator. In practice, the MATLAB (1997) function, \texttt{randn} , is used in this work.

A realisation of the stochastic input process is obtained by the following marginal transformation, (Kiuhegan and Liu, 1986)

\[
x_i(t_k) = F_{X_i}^{-1}(\Phi(z_i(t_k))) \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, M
\]

where:
- \( F_{X_i} \) = Distribution function for \( X_i(t) \) (n.d.)
- \( \Phi \) = Standard normal distribution, i.e. normal distribution with zero mean and standard deviation one
- \( z_i \) = Realisation of component of dependent standard normal vector \( z \) (n.d.)

The dependency between the components of \( x \), mutually and in time, is given by the correlation coefficient matrix, \( \rho_{XX'} \), of the generalised vector, \( x^* = [x(t_1)^\top \ldots x(t_M)^\top]^\top \), which is written using the temporal discretisation as:
The elements of the correlation coefficient function matrix are the values of auto-correlation functions \( \rho_{X_iX_i} \) (elements) and cross-correlation functions (other elements) corresponding to the different combination of components of the \( x \)-vector and between different time steps in the time series. In the present work, the following expression for the auto/cross correlation function is used

\[
\rho_{X_iX_j}(t_k, t_l) = \rho_{X_iX_j}(\tau) = \rho_{X_iX_j}(0) \cdot \exp \left[ -\frac{|\tau|}{a_{ij}} \right] \quad (2.3)
\]

where:
- \( \tau \) = Time-lag, \( \tau = t_k - t_l \)
- \( \rho_{X_iX_j}(0) \) = Correlation corresponding to zero time-lag
- \( a_{ij} \) = Correlation time corresponding to components \( i \) and \( j \).

An approximate relation between the correlation coefficient matrix, \( \rho_{XX} \), for the original discretised vector process, and the correlation coefficient matrix, \( \rho_{ZZ} \), for the dependent normal distributed vector process is given by, see Kiureghian and Liu, 1986

\[
\rho_{X_iX_j}(t_k, t_l) = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varphi_2 \left( z_i(t_k), z_j(t_l), \rho_{Z_iZ_j}(t_k, t_l) \right) \frac{1}{\sigma_{X_i} \sigma_{X_j}} \exp \left[ -\frac{\left( z_i(t_k) - \mu_{X_i} \right)^2}{\sigma_{X_i}^2} - \frac{\left( z_j(t_l) - \mu_{X_j} \right)^2}{\sigma_{X_j}^2} \right] dz_i dz_j
\]

where \( \varphi_2 \) is the two-dimensional normal distribution function given by:

\[
\varphi_2 \left( z_i(t_k), z_j(t_l), \rho_{Z_iZ_j}(t_k, t_l) \right) = \frac{1}{2 \pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ \frac{-z_i(t_k)^2 - 2 \rho_{Z_iZ_j}(t_k, t_l) z_j(t_l) z_i(t_k) + z_j(t_l)^2}{2 \left( 1 - \rho_{Z_iZ_j}(t_k, t_l)^2 \right)} \right] \frac{1}{\sqrt{1 - \rho_{Z_iZ_j}(t_k, t_l)^2}} dz_i dz_j
\]

Equation (2.4) is solved numerically for \( \rho_{Z_iZ_j}(t_k, t_l) \), for details see Brohus et al. 2002a.

What remains to be determined is a relation between the realisation of the dependent standard normal vector, \( z \), and the realisation of the independent standard normal variables, \( u \). This is achieved by the following transformation:

\[
z^* = T u^*
\]

where \( z^* = [z(t_1)^T \ldots z(t_{M})^T]^T \) and \( u^* = [u(t_1)^T \ldots u(t_{M})^T]^T \) can be regarded as generalised vectors and where \( T \) is a lower triangular matrix. The components are given by
Equation (2.7) is solved by Cholesky decomposition; see for instance (Press et al., 1989).

For large time series, the dimension of \( \rho_{xx} \) becomes very large and the transformation is difficult. However, it is possible to adapt a narrow bandwidth formulation and neglect terms, \( \rho_{x,t_i}(t_k,t_l) \), that are below a given value in the transformation. In practise, this is achieved by considering reduced versions of the vectors, \( x \), \( z \) and \( u \), where only components corresponding to a given number of the previous time steps are included, thus neglecting the insignificant dependency of all previous realisations.

For small values of the time lag, \( \tau \), the correlation matrix is not positive definite, making the Cholesky decomposition in equation (2.7) impossible. Thus, the time lag has a lower bound. At a first glance it seems to introduce problems when small time increments are to be used, e.g. when the simulated results are used in single and multizone simulation programs. However, if this is the case, the realisations of the stochastic processes at neighbouring time increments are so dependent on each other, that they will be almost the same. Thus, when smaller time increments are needed for single and multizone simulation, linear interpolation is conducted between the simulated values of the stochastic processes.

If the input parameters are time independent, more simple expressions are obtained. Thus, equation (2.6) is replaced with

\[
z(t_k) = T u(t_k) \quad , \quad k = 1, 2, \ldots, M
\]  

(2.8)

The lower triangular matrix, \( T \), is again obtained by equation (2.7), however it is possible to specify \( \rho_{xx} \) by the point-wise correlation coefficients and solve equation (2.8) for each time step separately

\[
\rho_{xx} = \rho_{xx} = \begin{bmatrix}
\rho_{x,x_1} & \cdots & \rho_{x,x_n} \\
\vdots & \ddots & \vdots \\
\rho_{x,x_1} & \cdots & \rho_{x,x_n}
\end{bmatrix}
\]  

(2.9)

The principle of the MCS when the parameters are mutually dependent and time independent is shown in Figures 2.3 and 2.4. The difference compared with the general time dependent case is that the input parameters can be simulated independently for each time step. In the general case, however, each time series must be simulated before the deterministic model can be used to obtain the output time series. This is due to the fact that realisations obtained from all previous time steps are used to obtain every new realisation. Thus, the computations in the time depending case become much more comprehensive in terms of CPU power and storage requirements.

The MCS approach used in the present report can be stated as:

1) Generate a realisation of an independent standard Normal distributed vector, \( u(t_k) \), \( k = 1, 2, \ldots, M \), e.g. by means of the MATLAB \texttt{randn} function.
2) Generate the stochastic input parameter time series, \( x(t_k), k = 1, 2, \ldots, M \), taking into account the dependency between the parameters (and in time) using the described Nataf transformation model, equations (2.1) – (2.7).

3) Calculate the output parameter time series, \( y(t_k), k = 1, 2, \ldots, M \), using the deterministic model (e.g. single or multizone model).

4) Repeat 1) – 3) \( N \) times.

5) Perform a statistical analysis of the results in order to calculate the time-varying distribution functions and statistics of the output quantities.
Figure 2.3 MCS approach. Generation of dependent input quantities at time $t$ when the parameters are mutually dependent and time independent.
Figure 2.4 MCS approach. Generation of dependent output time series from realisations of input time series.
The following section shows how the statistics and distribution functions of the output parameters can be obtained by statistical sampling among the simulated output time series.

2.2 Determination of output distribution functions based on MCS

The elements of the time-varying mean value function, $\mu_y(t)$, can be estimated by statistical sampling on the realisations of the output processes (Ross, 1987)

$$\mu_{y_i}(t_k) \approx \frac{1}{N} \sum_{r=1}^{N} y_{i,r}(t_k)$$  \hspace{1cm} (2.10)

The elements of the cross-covariance function, $\text{Cov}_{yy}(\tau)$, are estimated by

$$\text{Cov}_{y_iy_j}(t_k,t_l) \approx \frac{1}{N} \sum_{r=1}^{N} y_{i,r}(t_k)y_{j,r}(t_l) - \mu_{y_i}(t_k)\mu_{y_j}(t_l)$$ \hspace{1cm} (2.11)

The standard deviation is $\sigma_{y_i}(t_k) = \sqrt{\text{Cov}_{y_iy_i}(t_k,t_k)}$, and the cross-correlation coefficient

$$\rho_{y_iy_j}(t_k,t_l) = \frac{\text{Cov}_{y_iy_j}(t_k,t_l)}{\sigma_{y_i}(t_k)\sigma_{y_j}(t_l)}$$ \hspace{1cm} (2.12)

The distribution function, $F_{y_i}(y_i(t_k))$, is defined as the following probability

$$F_{y_i}(y_i(t_k)) = P(y_i(t_k) \leq y_i(t_k))$$ \hspace{1cm} (2.13)

For a given time step, $t_k$, the data for each output variable, $y_{i,r}(t_k)$, $r = 1, 2, \ldots, N$, are sorted in increasingly order, making it possible to determine $F_{y_i}(y_i(t_k))$ by

$$F_{y_i}(y_{i,r}(t_k)) \approx \frac{\text{number of } y_i(t_k) \leq y_{i,r}(t_k)}{N}$$ \hspace{1cm} (2.14)

In the following chapters, the MCS approach will be applied to estimate the statistics of the air flow in buildings, based on single zone and multizone air flow models.

3 Single zone model

In this chapter, a single zone model of a hybrid-ventilated enclosure is presented. The natural driving forces due to wind and air density differences are used to drive air through the building. In order to maintain an acceptable indoor air quality, a target air flow rate is required. When the air flow due to the natural driving forces is too low, a fan situated at the top of the building is started, and controlled to keep the target air flow. If the natural air flow too small, the fan is stopped and a damper situated at the west façade is regulated to maintain the target air flow.
Using the MCS approach, multiple time series of air flow and control parameters are derived from simulated time series of the input loads using the single zone model. Time-varying statistics, i.e., mean values, standard deviations and distribution functions are then obtained from the simulated results using statistical sampling. A test case is examined in order to illustrate the approach.

### 3.1 Single zone building model

A simple hybrid ventilated enclosure is investigated as shown in Figure 3.1. $v_{ext}(t)$ is the wind speed at the so-called reference height. Here $t$ is the time, denoting that the wind speed is a time-varying process. The outdoor climate is further described by the external air temperature, $\theta_{ext}(t)$, and the water humidity ratio of the external air, $W_{ext}(t)$. The indoor climate is described by the internal air temperature, $\theta_{int}(t)$, and the water humidity ratio, $W_{int}(t)$. The front opening of the building is situated at the height, $h_1$, above ground level and the top opening at height, $h_2$, above ground level. The atmospheric pressure at ground level is $P_{atm}$ outside the enclosure.

![Figure 3.1 Single zone model of hybrid ventilated enclosure.](image)

In order to maintain a specified air flow, $Q_{req}$, two control mechanisms are supplied. If the natural driving forces due to wind pressure, density and height differences are too large a damper on the front of the building (left) reduces the air flow. The damper is described by the opening angle, $s(t)$. On the other hand, if the air flow is too small, a fan is started to increase the air flow by means of mechanical driving forces. The fan is controlled by electrical power, $F(t)$. The damper is only used when the fan is off. Similarly, the fan is running only when the damper is fully opened.

### 3.2 Flow equation

In the following an equation is derived, relating the air flow, $Q(t)$, to the quantities $v_{ext}(t)$, $\theta_{ext}(t)$, $\theta_{int}(t)$, $W_{int}(t)$, $W_{ext}(t)$, $s(t)$ and $F(t)$. By solving the equation for the air flow at time $t$, it is possible to select control parameters, $s$ and $F$, for the next time increment to either decrease or increase the air flow towards a requested value, $Q_{req}$.

The absolute pressure outside the building at height, $h_1$, corresponding to the front opening, when assuming a hydrostatic pressure distribution, is given by
\[ P_{\text{ext}}^{\text{front}} = P_{\text{atm}} - \rho_{\text{ext}} g h_1 + \Delta P_{\text{wind}}^{\text{front}} \]  

where: 
\[ \rho_{\text{ext}} \] = Density of external air (kg/m\(^3\))  
\[ g \] = Gravitational acceleration (g = 9.82 m/s\(^2\))  
\[ \Delta P_{\text{wind}}^{\text{front}} \] = Wind pressure outside the front opening (Pa)

The air density, \( \rho \), is estimated by the following expression, (COMIS, 1990)

\[ \rho = \frac{P_{\text{atm}} (1 + W)}{461.518(\theta + 273.15)(W + 0.62198)} \text{ (kg/m}^3\text{)} \]  

where: 
\[ W \] = Water humidity ratio (kg/kg)  
\[ \theta \] = Air temperature (°C)

As the parameters \( W \) and \( \theta \) are assumed constant in space inside and outside the zone, respectively, the internal air density and the external air density will be constant in space, too.

The absolute pressure outside the building at height, \( h_2 \), corresponding to the top opening is given by

\[ P_{\text{ext}}^{\text{top}} = P_{\text{atm}} - \rho_{\text{ext}} g h_2 + \Delta P_{\text{wind}}^{\text{top}} \]  

where \( \Delta P_{\text{wind}}^{\text{top}} \) is the wind pressure outside the top opening.

The absolute pressure inside the two openings is given by, respectively

\[ P_{\text{int}}^{\text{front}} = P - \rho_{\text{int}} g h_1 \]  

\[ P_{\text{int}}^{\text{top}} = P - \rho_{\text{int}} g h_2 \]  

where: 
\[ P \] = Absolute pressure within the enclosure at ground level (Pa)  
\[ \rho_{\text{int}} \] = Density of internal air (kg/m\(^3\))

The pressure difference over the front opening must balance the flow resistance over the damper, i.e.

\[ \Delta P_{\text{damp}} = P_{\text{ext}}^{\text{front}} - P_{\text{int}}^{\text{front}} = P_{\text{atm}} - P + \Delta P_{\text{wind}}^{\text{front}} + (\rho_{\text{int}} - \rho_{\text{ext}}) g h_1 \]  

Similarly, the pressure difference over the top opening, must balance the pressure increase over the fan

\[ \Delta P_{\text{mech}} = P_{\text{ext}}^{\text{top}} - P_{\text{int}}^{\text{top}} = P_{\text{atm}} - P + \Delta P_{\text{wind}}^{\text{top}} + (\rho_{\text{int}} - \rho_{\text{ext}}) g h_2 \]  

By elimination of the pressure, \( P \), between equation (3.6) and equation (3.7) the following expression is obtained
\[ \Delta P_{\text{wind}}^{\text{front}} - \Delta P_{\text{wind}}^{\text{top}} + (\rho_{\text{ext}} - \rho_{\text{int}})g(h_2 - h_1) + \Delta P_{\text{mech}} = \Delta P_{\text{damp}} \]

\[ \Delta P_{\text{wind}} + \Delta P_{\text{stack}} + \Delta P_{\text{mech}} = \Delta P_{\text{damp}} \]  

(3.8)

where \( \Delta P_{\text{wind}} = \Delta P_{\text{wind}}^{\text{front}} - \Delta P_{\text{wind}}^{\text{top}} \) is the wind pressure difference and \( \Delta P_{\text{stack}} = (\rho_{\text{ext}} - \rho_{\text{int}})g\Delta h \) is the stack pressure difference over the building envelope.

Equation (3.8) states that the driving forces due to wind, \( \Delta P_{\text{wind}} \), stack effect, \( \Delta P_{\text{stack}} \), and fan power, \( \Delta P_{\text{mech}} \), must balance the resistance through the system, represented by the damper, \( \Delta P_{\text{damp}} \). \( \Delta P_{\text{mech}} \) and \( \Delta P_{\text{damp}} \) can be expressed as functions of the air flow through the zone, which is implicitly assumed to be constant through all components and, thus, is the basis for the apparent “summation” encountered in equation (3.8). Afterwards the resulting nonlinear equation can be solved numerically for the air flow, see Chapter 3.7. An alternative derivation of the relationship - taking its stating point in a mass balance (mass conservation) - is shown in Appendix A, leading to an identical solution.

Chapter 3.3 describes how the wind pressure term, \( \Delta P_{\text{wind}} \), is obtained, in Chapter 3.4 a model for the fan pressure increase, \( \Delta P_{\text{mech}} \), is stated, in Chapter 3.5 a model of the damper pressure decrease, \( \Delta P_{\text{damp}} \), is given and in Chapter 3.6 it is described how the nonlinear flow equation can be solved for the air flow, \( Q \), using Newton’s method.

### 3.3 Wind pressure difference

The wind pressure differences corresponding to the front and the top openings are given by Haghighat and Rao (1991)

\[ \Delta P_{\text{wind}}^{\text{front}} = \frac{1}{2}\rho_{\text{ext}}C_P^{\text{front}} v^2 \]  

(3.9)

\[ \Delta P_{\text{wind}}^{\text{top}} = \frac{1}{2}\rho_{\text{ext}}C_P^{\text{top}} v^2 \]  

(3.10)

where:  
\( C_P^{\text{front}} = \) Pressure coefficient for front opening (n.d.)  
\( C_P^{\text{top}} = \) Pressure coefficient for top opening (n.d.)  
\( v = \) Wind speed, for actual building height and terrain roughness (m/s)

The pressure coefficients, \( C_P^{\text{front}} \) and \( C_P^{\text{top}} \), are functions of the wind direction and the wind speed.

The following wind profile (Grosso, 1992) is used to model the wind acting on the building envelope

\[ v = v_{\text{ext}} \left( \frac{H}{z_0} \right)^\alpha \]  

(3.11)

where:  
\( z_0 = \) Reference height for the wind profile (m), typical \( z_0 = 10 \) m  
\( \alpha = \) Exponent in the wind profile, depending on the terrain roughness (n.d.)
Thus, the wind pressure difference between the openings can be obtained from equations (3.9) and (3.10) to

\[
\Delta P_{\text{wind}} = \frac{1}{2} \rho_{\text{ext}} \left( C_{p}^{\text{front}} - C_{p}^{\text{top}} \right) v^2
\]

(3.12)

3.4 Fan model

The pressure difference over the fan must be modelled by an analytical expression in order to use it together with the air flow model. However, the fan manufacturers often specify the properties of fans operating at a limited number of conditions. Therefore, interpolation is conducted in order to model the fan at all conditions. In the following, it is shown how such an analytical expression can be obtained from a data chart.

The pressure increase over the fan, \(\Delta P_{\text{mech}}\), is a function of the air flow, \(Q\), the fan power supply, \(F\), and the density, \(\rho_{\text{int}}\), of the room air passing the fan.

The room air density, \(\rho_{\text{int}}\), can be expressed as a function of the room air temperature, \(\theta_{\text{int}}\), the room water humidity ratio, \(W_{\text{int}}\), and the atmospheric pressure, \(P_{\text{atm}}\), by equation (3.2). Thus, the fan pressure difference, \(\Delta P_{\text{mech}}\), can be regarded as a function of air flow, power supply, internal air temperature and water humidity ratio, such that \(\Delta P_{\text{mech}} = \Delta P_{\text{mech}}(Q, F, \theta_{\text{int}}, W_{\text{int}})\).

In this study, data for a fan operating at zero and maximum power supplies has been used (Landbrugsministeriet, 1992). No information has been provided regarding the air temperature and water humidity ratio in these cases. The data are assumed to be given for the fan operating with dry air, i.e. \(W_{\text{int}} = 0 \text{ kg/kg}\) and \(\theta_{\text{int}} = 20 \degree C\). Thus, the first step is to estimate the pressure difference over the fan operating at these conditions, and then to consider the general case latter. In Tables 3.1 and 3.2, and in Figure 3.3 are shown data for the fan.

<table>
<thead>
<tr>
<th>Table 3.1</th>
<th>Relation between air flow, (Q_{\text{min}}), and pressure difference, (\Delta P_{\text{mech}}^{\text{min}}), over stopped fan, (F_{\text{min}} = 0 \text{ W}), for (\theta_{\text{int}} = 20 \degree C) and (W_{\text{int}} = 0 \text{ kg/kg}). Data from Landbrugsministeriet (1992).</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_{\text{min}}) (m³/s)</td>
<td>0.106</td>
</tr>
<tr>
<td>(\Delta P_{\text{mech}}^{\text{min}}) (Pa)</td>
<td>-3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.2</th>
<th>Relation between air flow, (Q_{\text{max}}), and pressure difference, (\Delta P_{\text{mech}}^{\text{max}}), over fan running at maximum power, (F_{\text{max}} = 90 \text{ W}), for (\theta_{\text{int}} = 20 \degree C) and (W_{\text{int}} = 0 \text{ kg/kg}). Data from Landbrugsministeriet (1992).</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_{\text{max}}) (m³/s)</td>
<td>0.000</td>
</tr>
<tr>
<td>(\Delta P_{\text{mech}}^{\text{max}}) (Pa)</td>
<td>85</td>
</tr>
</tbody>
</table>

When the fan is off, it can be regarded as a resistance to the air flow corresponding to a negative pressure increase, \(\Delta P_{\text{mech}}^{\text{min}}\). In this case, the relation between the air flow and pressure difference is modelled by the following air flow equation, adapted from an equation for flow through an opening, see Feustel and Dieris (1992).
\[ Q_{min}^{\text{min}} = A_1 \left( - \Delta P_{\text{mech}}^{\text{min}} \right)^{1/2} \]  
(3.13)

where:  
\( A_1 \) = Coefficient (m\(^3\)/Pa·s)  
\( A_2 \) = Coefficient (n.d.)

Equation (3.13) can be rewritten as

\[ \Delta P_{\text{mech}}^{\text{min}} \left( Q_{min}^{\text{min}} \right) = \left( \frac{Q_{min}^{\text{min}}}{A_1} \right)^{1/2} \]  
(3.14)

The coefficients \( A_1 \) and \( A_2 \) are obtained as follows. By taking the logarithm of equation (3.13) one obtains

\[ \log(Q_{min}^{\text{min}}) = \log(A_1) + A_2 \log(- \Delta P_{\text{mech}}^{\text{min}}) \]  
(3.15)

\( A_1 \) and \( A_2 \) are then obtained by linear regression of \( \log(Q_{min}^{\text{min}}) \) on \( \log(- \Delta P_{\text{mech}}^{\text{min}}) \), using the regression equation (3.15) and the data in Table 3.1.

The data points \((Q_{1, \text{min}}, \Delta P_{\text{mech}, 1}^{\text{min}}), (Q_{2, \text{min}}, \Delta P_{\text{mech}, 2}^{\text{min}}), (Q_{3, \text{min}}, \Delta P_{\text{mech}, 3}^{\text{min}})\) are assembled in following matrices

\[
\begin{bmatrix}
1 & \log(- \Delta P_{\text{mech}, 1}^{\text{min}}) \\
\vdots & \vdots \\
1 & \log(- \Delta P_{\text{mech}, 3}^{\text{min}})
\end{bmatrix}
\quad \begin{bmatrix}
\log(Q_{1, \text{min}}) \\
\vdots \\
\log(Q_{3, \text{min}})
\end{bmatrix}
\]  
(3.16)

It is possible to obtain an unweighted least squares estimator of the coefficients, \( A = [A_1 \ A_2]^T \), according to Myers and Montgomery (1995) by

\[
A_i = \exp(A_i^*) \quad A_2 = A_2^*
\]  
(3.17)

where:

\[
A^* = (X_A^T X_A)^{-1} X_A^T y_A
\]  
(3.18)

By conducting the analysis, the values \( A_1 = 0.0428 \) (m\(^3\)/Pa·s) and \( A_2 = 0.8585 \) (n.d) are obtained.

The pressure increase at maximum power supply, \( \Delta P_{\text{mech}, \text{max}} \), is modelled by the following parabolic equation, see Feustel and Dieris (1992)

\[ \Delta P_{\text{mech}}^{\text{max}} \left( Q_{\text{max}} \right) = B_1 + B_2 Q_{\text{max}}^{\text{max}} + B_3 Q_{\text{max}}^{\text{max}}^2 \]  
(3.19)

where:  
\( B_1 \) = Coefficient (Pa)  
\( B_2 \) = Coefficient (Pa·s/m\(^3\))  
\( B_3 \) = Coefficient (Pa·s\(^2\)/m\(^6\))
The coefficients, $B_1$, $B_2$ and $B_3$, are obtained by a regression of $\Delta P_{\text{mech}}^{\text{max}}$ on $Q^{\text{max}}$, using the regression equation (3.19) and the data in Table 3.2.

By assembling the data points $(Q_1^{\text{max}}, \Delta P_{\text{mech},1}^{\text{max}}), \ldots, (Q_{11}^{\text{max}}, \Delta P_{\text{mech},11}^{\text{max}})$ in following matrices

$$\begin{bmatrix} 1 & Q_1^{\text{max}} & \left(Q_1^{\text{max}}\right)^2 \\ \vdots & \vdots & \vdots \\ 1 & Q_{11}^{\text{max}} & \left(Q_{11}^{\text{max}}\right)^2 \end{bmatrix}, \quad \begin{bmatrix} \Delta P_{\text{mech},1}^{\text{max}} \\ \vdots \\ \Delta P_{\text{mech},11}^{\text{max}} \end{bmatrix}$$

an unweighted least squares estimator of the coefficients, $B=[B_1 \ B_2 \ B_3]^T$, is obtained by:

$$B = (X_B^T X_B)^{-1} X_B^T y_B$$

The resulting coefficients become $B_1 = 82.5435$ Pa, $B_2 = -205.8038$ Pa·s/m³ and $B_3 = 69.0924$ Pa·s²/m⁶.

A relation for the pressure increase, $\Delta P_{\text{mech}}^{\text{int}}$, over the fan at an “intermediate” power supply, $F^{\text{int}}$, is derived using the following approximate relations, i.e. the fan laws (Jones, 1992).

$$\frac{Q^{\text{int}}}{Q^{\text{max}}} = \frac{n^{\text{int}}}{n^{\text{max}}}$$

$$\frac{\Delta P_{\text{mech}}^{\text{int}}}{\Delta P_{\text{mech}}^{\text{max}}} = \left(\frac{n^{\text{int}}}{n^{\text{max}}}\right)^2$$

$$\frac{F^{\text{int}}}{F^{\text{max}}} = \left(\frac{n^{\text{int}}}{n^{\text{max}}}\right)^3$$

where: $Q^{\text{int}} = \text{Air flow at intermediate power supply (m}^3/\text{s})$

$n^{\text{int}} = \text{Number of revolutions pr. time for intermediate power supply (n.d.)}$

$n^{\text{max}} = \text{Number of revolutions pr. time for maximum power supply (n.d.)}$

In this work, $F^{\text{int}} / F^{\text{max}} = 0.75$, is used in order to select an “intermediate” power supply nearer to the maximum than the minimum power supply, as equations (3.22) – (3.24) are valid for a running fan and not the situation at minimum power, where the fan is stopped.

By insertion of equations (3.22) and (3.24) into equation (3.23) one obtains

$$\Delta P_{\text{mech}}^{\text{int}}\left(Q^{\text{int}}\right) = \Delta P_{\text{mech}}^{\text{max}}\left(Q^{\text{max}}\right) \cdot \left(\frac{n^{\text{int}}}{n^{\text{max}}}\right)^2 = \Delta P_{\text{mech}}^{\text{max}}\left(Q^{\text{int}} \cdot \frac{n^{\text{max}}}{n^{\text{int}}} \cdot \frac{n^{\text{int}}}{n^{\text{max}}}\right)^2$$

$$= \Delta P_{\text{mech}}^{\text{max}}\left(Q^{\text{int}} \left(F^{\text{max}} / F^{\text{int}}\right)^{1/3}\right)^2 \cdot \left(F^{\text{int}} / F^{\text{max}}\right)^{2/3}$$

(3.25)
By inserting the expression (3.19) for $\Delta P_{\text{mech}}^{\max}$ into equation (3.25) the following expression is obtained

$$\Delta P_{\text{mech}}(Q) = C_1 + C_2 Q + C_3 (Q)^2$$

(3.26)

where

$$C_1 = B_1 \left( \frac{F_{\text{int}}}{F_{\text{max}}} \right)^{2/3}$$

(3.27)

$$C_2 = B_2 \left( \frac{F_{\text{int}}}{F_{\text{max}}} \right)^{1/3}$$

(3.28)

$$C_3 = B_3$$

(3.29)

The resulting coefficients become $C_1 = 68.1382 \text{ Pa}$, $C_2 = -186.9852 \text{ Pa} \cdot \text{s/m}^3$ and $C_3 = 69.0924 \text{ Pa} \cdot \text{s}^2/\text{m}^6$.

A second order polynomial is introduced to model the fan at a general power supply, $F$

$$\Delta P_{\text{mech}}(Q, F) = D_1(Q) + D_2(Q)F + D_3(Q)F^2$$

(3.30)

where:

$$D_1(Q) = \Delta P_{\text{mech}}^{\min}(Q)$$

(3.31)

$$D_2(Q) = \frac{F_{\text{max}} \cdot \Delta P_{\text{mech}}^{\min}(Q)}{F_{\text{int}} \left( F_{\text{max}} - F_{\text{int}} \right)} - \frac{F_{\text{int}} \cdot \Delta P_{\text{mech}}^{\max}(Q)}{F_{\text{max}} \left( F_{\text{max}} - F_{\text{int}} \right)} - \frac{(F_{\text{max}} + F_{\text{int}}) \Delta P_{\text{mech}}^{\min}(Q)}{F_{\text{max}} F_{\text{int}}}$$

(3.32)

$$D_3(Q) = \frac{\Delta P_{\text{mech}}^{\min}(Q)}{F_{\text{max}} F_{\text{int}}} + \frac{\Delta P_{\text{mech}}^{\max}(Q)}{F_{\text{max}} \left( F_{\text{max}} - F_{\text{int}} \right)} - \frac{\Delta P_{\text{mech}}^{\int}(Q)}{F_{\text{int}} \left( F_{\text{max}} - F_{\text{int}} \right)}$$

(3.33)

The basic idea of the interpolation scheme for the fan pressure difference, $\Delta P_{\text{mech}}$, is shown in Figure 3.2. The air flow, e.g. $Q_1$, is used to obtain corresponding values of minimum, $\Delta P_{\text{mech}}^{\min}$, intermediate, $\Delta P_{\text{mech}}^{\int}$, and maximum pressure difference, $\Delta P_{\text{mech}}^{\max}$, (left part of the figure). Then the pressure difference, $\Delta P_{\text{mech}}$, corresponding to the actual power supply, $F$, is obtained using a second order polynomial interpolation (right part of the figure).
Figure 3.2 Determination of pressure difference, $\Delta P_{\text{mech}}$, over fan as a function of air flow, $Q$, and power supply, $F$, shown for two different air flows, $Q_1$ and $Q_2$.

The expression for the fan pressure difference, $\Delta P_{\text{mech}}$, given above is valid for a room air temperature, $\theta_{\text{int}} = 20 \, ^\circ\text{C}$ and a water humidity ratio, $W_{\text{int}} = 0 \, \text{kg/kg}$. Relations this condition (denoted with an apostrophe ‘) and an arbitrary case are given by (Sørensen, 1988)

\[ Q' = Q \frac{\rho}{\rho'} \]  
\[ F' = F \frac{\rho}{\rho'} \]  
\[ \Delta P'_{\text{mech}} = \Delta P_{\text{mech}} \frac{\rho'}{\rho} \]

(3.34)  
(3.35)  
(3.36)

It can be observed from equations (3.34) – (3.36) that if the air density, $\rho$, does not vary much from the air density corresponding to the case that the manufacturer has specified, $\rho'$, the resulting air flow, power supply and pressure difference is almost the same, and the equations (3.34-3.36) can be omitted.
Thus, the pressure difference over the fan, $\Delta P_{\text{mech}} = \Delta P_{\text{mech}}(Q, F, \theta_{\text{int}}, W_{\text{int}})$, is in general calculated by the following procedure.

- Air flow, $Q'$, and power supply, $F'$, are calculated by equations (3.34) and (4.35) respectively.

- Pressure differences, $\Delta P_{\text{mech}}^{\text{min}}$, $\Delta P_{\text{mech}}^{\text{int}}$ and $\Delta P_{\text{mech}}^{\text{max}}$ are calculated by equations (3.14), (3.26) and (3.19) respectively as functions of $Q'$ and $F'$.

- Pressure difference, $\Delta P_{\text{mech}}'$, is calculated by the second order polynomial approximation, equation (3.30).

- Pressure difference, $\Delta P_{\text{mech}}$, is calculated by equation (3.36).

Figure 3.3 shows a plot of the pressure difference, $\Delta P_{\text{mech}}$, as a function of the air flow, $Q$, for different power supplies, $F$, corresponding to $\theta_{\text{int}} = 20 \, ^{\circ}\text{C}$ and $W_{\text{int}} = 0 \, \text{kg/kg}$.

![Figure 3.3](image_url)

**Figure 3.3** Modelled fan behaviour shown together with the data for minimum fan power (triangles) and for maximum fan power (squares).

### 3.5 Damper model

The damper is introducing a resistance to the air flow, which can be controlled by the opening angle, $s$, of the damper.
The pressure decrease (defined positive) over the damper at a given damper opening angles, \( s \), is modelled by a second order polynomial, see Kjerulf-Jensen, 1989, assuming that the pressure drop over the damper is independent of the air density

\[
\Delta P_{\text{damp}}(s, Q) = R(s)Q^2
\]  

(3.37)

where the damper resistance, \( R(s) \), as a function of the damper opening angle is modelled by

\[
R(s) = D_1 s^{D_2}
\]  

(3.38)

where:

\[
D_1 = \text{Coefficient (Pa·s}^2/m^6) \\
D_2 = \text{Coefficient (n.d.)}
\]

Equation (3.38) has been chosen by observing data from Kjerulf-Jensen, 1989. The resulting parameters, \( D_1 \) and \( D_2 \), for a damper with a cross section area of 0.088 m\(^2\), are shown in Table 3.3.

### Table 3.3 Data for damper operating at \( \Delta P_{\text{damp}} = 5 \) Pa, (Kjerulf-Jensen, 1989).

<table>
<thead>
<tr>
<th>( s ) (°)</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q ) (m(^3)/s)</td>
<td>0.0002</td>
<td>0.0048</td>
<td>0.0229</td>
<td>0.0616</td>
<td>0.1320</td>
<td>0.2640</td>
<td>0.4840</td>
</tr>
</tbody>
</table>

By inserting equation (3.37) into equation (3.38) and taking the logarithm, one obtains

\[
\log \left( \frac{\Delta P_{\text{damp}}}{Q^2} \right) = \log(D_1) + D_2 \log(s)
\]  

(3.39)

The coefficients, \( D_1 \) and \( D_2 \), are obtained by linear regression of \( \log(\Delta P_{\text{damp}}/Q^2) \) on \( \log(s) \), using the regression equation (3.39) and the data in Table 3.3.

By assembling the data points \((s_1, Q_1^2)\)...\((s_7, Q_7^2)\) in the following matrices

\[
X_D = \begin{bmatrix} 1 & \log(s_1) \\ \vdots & \vdots \\ 1 & \log(s_7) \end{bmatrix}, \quad y_D = \begin{bmatrix} \log \left( \frac{\Delta P_{\text{damp}}}{Q_1^2} \right) \\ \vdots \\ \log \left( \frac{\Delta P_{\text{damp}}}{Q_7^2} \right) \end{bmatrix}
\]  

(3.40)

a unweighted least squares estimator of the coefficients, \( D=[D_1 \ D_2]^T \), is obtained by

\[
D_1 = \exp(D_1^*) , \quad D_2 = D_2^*
\]  

(3.41)

where

\[
D^* = (X_D^TX_D)^{-1}X_D^Ty_D
\]  

(3.42)

By conducting the analysis, \( D_1 = 2.66 \cdot 10^{11} \) Pa·s\(^2\)/m\(^6\) and \( D_2 = -5.0916 \) (n.d.) are obtained.
Figure 3.4 shows plots of the pressure decrease, $\Delta P_{damp}$, as a function of the air flow, $Q$, for different opening angles, $s$, together with the data from Table 3.3. A reasonable agreement is observed.

![Figure 3.4 Modelled damper behaviour shown together with the data from Table 4.3 (crosses).](image)

3.6 Control strategy

As mentioned previously, the damper and the fan are controlled to maintain a target air flow, $Q_{req}$. In order to avoid control instability and reduce equipment exposure to hard wear, the air flow is allowed to vary freely within a dead band (Jones, 1992).

$$\left(1 - Q_{fac}\right)Q_{req} \leq Q(t) \leq \left(1 + Q_{fac}\right)Q_{req}$$  \hspace{1cm} (3.43)

where $Q_{fac}$ is a factor determining the width of the dead band (n.d.)

Thus, the control parameters, $s(t)$ and $F(t)$, are adjusted only when the air flow is outside of the selected dead band. If $Q(t) > (1+Q_{fac})Q_{req}$ the air flow is too large and the control parameters are adjusted to insure an air flow within the dead band. On the other hand, if $Q(t) < (1-Q_{fac})Q_{req}$ the air flow is too small and the control parameters are adjusted accordingly. Figure 3.5 shows the dead band in a principal sketch of the $Q(t)$ variation.
Figure 3.5 Control is performed only when the air flow is outside a specified dead band.

The case where the damper is fully opened and the fan is off can be regarded as a limit state between damper and fan operation. If the air flow is too large in this case, the opening angle of the damper, $s(t)$, is reduced in order to reduce the air flow. On the other hand, if the air flow is too small, more electric power, $F(t)$, is supplied to the fan in order to increase the air flow. In a situation where the damper is partly closed and the fan is off, control is conducted by either opening or closing the damper to increase or decrease the air flow. If the fan is already running and the damper is fully open, the power supply to the fan is adjusted. Thus, for each time increment only one of the parameters, $F(t)$ and $s(t)$, is adjusted while the other parameter is maintained, corresponding to either the damper opened or the fan is off. Both damper and fan can only be controlled between acceptable limits, i.e.

$$s_{\text{min}} \leq s(t) \leq s_{\text{max}}$$  \hspace{1cm} (3.44)

$$F_{\text{min}} \leq F(t) \leq F_{\text{max}}$$  \hspace{1cm} (3.45)

In this work, $s_{\text{min}} = 0^\circ$ and $s_{\text{max}} = 90^\circ$, $F_{\text{min}} = 0$ W and $F_{\text{max}} = 90$ W are chosen.

Both damper and fan are proportional plus integral controlled (PI); Only the expression for the damper is shown (Jones, 1992)

$$\Delta s(t) = K_{k,s} \left( \Delta Q(t) + \frac{1}{\tau_{I,s}} \int_{t_{b,s}}^{t} \Delta Q(t') dt' \right)$$  \hspace{1cm} (3.46)

where:  
- $K_{k,s}$ = Proportional factor for damper ($^\circ$/s/m$^3$)
- $\tau_{I,s}$ = Integration time for damper (m$^3$/$^\circ$)
- $t_{b,s}$ = Start time of damper regulation ($^\circ$)
- $\Delta Q$ = $Q_{\text{req}} - Q$

Both $\Delta s$ and $\Delta F$ are adjusted accordingly if the resulting opening degree and power supply are outside the bounds stated in equations (3.44) and (3.45).

The following control algorithm is adapted to control the air flow:

1) Select initial control settings (e.g. $s(t_0) = s_{\text{max}}^0$, $F(t_0) = F_{\text{min}}^0$, $t_{b,s} = t_{b,F} = t_0$).
2) Initiate time loop variable, \( k = 0 \).

3) Calculate air flow, \( Q(t_k) \), by Newton’s method.

4) Adjust control parameters, \( s(t_{k+1}) \) and \( F(t_{k+1}) \), according to:

\[
\text{If } Q(t_k) < (1-Q_{\text{fac}}) Q_{\text{req}} \text{ or } Q(t_k) > (1+Q_{\text{fac}}) Q_{\text{req}} \text{ perform control to change air flow:}
\]

\[
\text{If } (s(t_k) = s_{\text{max}} \land F(t_k) > F_{\text{min}}) \text{ change fan power:}
\]

\[
s(t_{k+1}) = s(t_k)
\]

\[
\Delta F(t_k) = K_{b,F} \left( \frac{1}{t_{t,F}} \int_{t_{b,F}}^{t_k} \Delta Q(t) dt \right)
\]

\[
F(t_{k+1}) = \min \left( F(t_k) + \Delta F, F_{\text{max}} \right)
\]

\[
F(t_{k+1}) = \max \left( F_{\text{min}}, F(t_{k+1}) \right)
\]

\[
t_{b,F} = t_k
\]

\[
\text{If } (s(t_k) \leq s_{\text{max}} \land F(t_k) = F_{\text{min}}) \text{ change damper opening angle:}
\]

\[
\Delta s(t_k) = K_{b,s} \left( \frac{1}{t_{t,s}} \int_{t_{b,s}}^{t_k} \Delta Q(t) dt \right)
\]

\[
s(t_{k+1}) = \min \left( s(t_k) + \Delta s, s_{\text{max}} \right)
\]

\[
s(t_{k+1}) = \max \left( s_{\text{min}}, s(t_{k+1}) \right)
\]

\[
F(t_{k+1}) = F(t_k)
\]

\[
t_{b,F} = t_k
\]

5) Update time loop variable, \( k = k + 1 \).

6) Goto 3) until last time step is reached.

The control of the damper and the fan are based on the time history of the air flow. Therefore only systematically varying external load processes or a time delay in the building physics, e.g. thermal capacity will ensure that the control parameters will adjust the damper and fan towards the target air flow in the model.

3.7 Solution of flow equation by Newton’s method

Equation (3.8) can be formulated as a nonlinear function of the air flow

\[
f(Q) = 0 \tag{3.47}
\]

The nonlinear equation can be solved by any standard method on nonlinear equation solving. Here, Newton’s method is used due to its simplicity. By making a first order Taylor series expansion of the function based on the point, \( Q_0 \), the following expression is obtained (Press et al., 1989)
By solving equation (3.48) for the unknown air flow, $Q$, one obtains

$$Q = Q_0 - \left( \frac{df}{dQ} \right)_{Q=Q_0}^{-1} f(Q_0)$$

(3.49)

The flow equation is then solved by selecting a starting value of the unknown air flow, $Q_0$, and equation (3.49) is used iteratively until a converged solution is obtained.

### 3.8 Test case

The week from January 1 to January 7 in the Danish Design Reference Year (DRY) (Jensen and Lund, 1995) has been selected for further analysis. In Table 3.4 are given various deterministic parameters defining the building and in Figure 3.6 are given the pressure coefficients, $C_{P_{front}}$ and $C_{P_{top}}$, given as functions of the wind direction, expressed as function of the wind direction measured clockwise from North, the data are adapted from Orme et al. 1998. The values of the parameters defining the PI – regulator, $K_{k,s}$, $K_{k,F}$, $\tau_{k,s}$ and $\tau_{k,F}$ have been adjusted in a pilot study to ensure reasonable stability of the regulator.

The front opening is situated on the west facade of the building.

#### Table 3.4 Deterministic parameters for hybrid ventilated building.

<table>
<thead>
<tr>
<th>$\Delta h$ (m)</th>
<th>$H$ (m)</th>
<th>$z_0$ (m)</th>
<th>$\alpha$ (n.d.)</th>
<th>$g$ (m/s²)</th>
<th>$P_{atm}$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
<td>0.28</td>
<td>9.82</td>
<td>101325</td>
</tr>
<tr>
<td>$\theta_{int}$ (°C)</td>
<td>$W_{int}$ (kg/kg)</td>
<td>$W_{ext}$ (kg/kg)</td>
<td>$Q_{req}$ (m³/s)</td>
<td>$K_{k,s}$= $K_{k,F}$ (°C/m³), (m³/°C)</td>
<td>$\tau_{k,s}$= $\tau_{k,F}$ (Ws/m³), (m³/W)</td>
</tr>
<tr>
<td>20</td>
<td>0.008</td>
<td>0.006</td>
<td>0.3</td>
<td>100</td>
<td>1000</td>
</tr>
</tbody>
</table>
Figure 3.6 $C_p$ values depending on wind direction (adapted from Orme et al. 1998)

The remaining parameters, i.e. $\theta_{ext}$, $v_{ext}$ and the corresponding wind direction are defined as stochastic quantities. In Brohus et al. (2002a), probabilistic weather models are presented; those parameters are derived from the Danish Design Reference Year (DRY), see (Jensen and Lund, 1995).

The external air temperature is modelled by a Gaussian distribution

$$
\begin{bmatrix}
    x_1(t_k) \\
    x_2(t_k) \\
    x_3(t_k)
\end{bmatrix} =
\begin{bmatrix}
    \theta_{ext}(t_k) \\
    v_{ext}(t_k) \\
    D(t_k)
\end{bmatrix} =
\begin{bmatrix}
    F^{-1}_{\theta_{ext}}(\Phi(y_1(t_k))) \\
    F^{-1}_{v_{ext}}(\Phi(y_2(t_k))) \\
    F^{-1}_{D}(\Phi(y_3(t_k)))
\end{bmatrix}
$$

(3.50)

The external air temperature is modelled by a Gaussian distribution

$$
F_{\theta_{ext}}(\theta_{ext}(t)) = \Phi\left(\frac{\theta_{ext}(t) - \mu_{\theta_{ext}}(t)}{\sigma_{\theta_{ext}}(t)}\right)
$$

(3.51)

with the following mean value function and standard deviation

$$
\mu_{\theta_{ext}}(t) = 7.76 + 8.93 \cos(2\pi f_1 t + 2.74) + 2.45 \cos(2\pi f_{365} t + 2.73) \quad (\text{°C})
$$

(3.52)

$$
\sigma_{\theta_{ext}} = 3.42 \quad (\text{°C})
$$

(3.53)

where: $t$ = Time from the beginning of the year, (s)

$f_1$ = Frequency corresponding to yearly variation, $f_1 = 1/(3600 \cdot 24 \cdot 365)$ Hz

$f_{365}$ = Frequency corresponding to daily variation, $f_{365} = 1/(3600 \cdot 24)$ Hz

The wind sector is described by the following discrete distribution

$$
F_D(D(t)) = \sum_{j=1}^{D(t)} P_{0,j}
$$

(3.54)
The wind direction is given by a conditional Weibull distribution (Pietrzyk, 1995):

\[
F_{v_{ext}|D}(v_{ext}) = P_0(D(t)) + (1 - P_0(D(t))) \left[ 1 - \exp\left( -\frac{v_{ext}(t)}{c(D(t))} \right) ^{\lambda(D(t))} \right]
\]  (3.55)

In Table 3.5 is given the parameters for the probability distributions for the wind speed and the wind sector.

<table>
<thead>
<tr>
<th>Wind Sector</th>
<th>Direction</th>
<th>Mean Angle (deg)</th>
<th>(P_0) (sector Zero probability) (n.d.)</th>
<th>(\lambda)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>North</td>
<td>0</td>
<td>0.132</td>
<td>1.562</td>
<td>4.1627</td>
</tr>
<tr>
<td>2</td>
<td>North-East</td>
<td>45</td>
<td>0.139</td>
<td>1.942</td>
<td>3.9213</td>
</tr>
<tr>
<td>3</td>
<td>East</td>
<td>90</td>
<td>0.117</td>
<td>2.071</td>
<td>5.5799</td>
</tr>
<tr>
<td>4</td>
<td>South-East</td>
<td>135</td>
<td>0.109</td>
<td>2.084</td>
<td>5.3617</td>
</tr>
<tr>
<td>5</td>
<td>South</td>
<td>180</td>
<td>0.125</td>
<td>2.020</td>
<td>4.2654</td>
</tr>
<tr>
<td>6</td>
<td>South-West</td>
<td>225</td>
<td>0.115</td>
<td>1.890</td>
<td>5.3237</td>
</tr>
<tr>
<td>7</td>
<td>West</td>
<td>270</td>
<td>0.116</td>
<td>1.961</td>
<td>6.7539</td>
</tr>
<tr>
<td>8</td>
<td>North-West</td>
<td>315</td>
<td>0.134</td>
<td>1.832</td>
<td>6.0248</td>
</tr>
</tbody>
</table>

The cross- and auto-correlation functions are given as equation (2.3). The parameters defining these functions are given in Table 3.6.

| \((x_i, x_j)\) \((\theta_{ext}, \theta_{ext})\) \((\theta_{ext}, D)\) \((\theta_{ext}, v_{ext})\) \((D, D)\) \((D, v_{ext})\) \((v_{ext}, v_{ext})\) \(a_{ij}\) (days) \(\rho_{a,ij}\) (n.d.) | 1.291 | 0.840 | 0.485 | 0.694 | 0.635 | 0.538 | 1.000 | 0.221 | 0.299 | 1.000 | 0.370 | 1.000 |

The transformation model of Chapter 2 is used to generate time series of the external air temperature. However, in order to deal with the conditional Weibull distribution for the wind speed, equation (3.55), each variable and each time step is generated in sequence, as previous realisation of the wind sector are needed in order to specify the conditional distribution. This is in practice accomplished by considering the generalised vector of input realisations, \(x^*\), to consist of only components up to the present time step and variable. Thus, the Nataf-transformation is conducted once for each element in the \(x^*\) - vector.

The model for the hybrid ventilated enclosure is now used to simulate time series of air flow, \(Q\), and control parameters, \(s\) and \(F\), using the simulated time series of external air temperature, wind speed and direction. (The direction is used to estimate the appropriate pressure coefficients) The result of such a simulation is shown in the Figures 3.7 – 3.9 using different values of \(Q_{fac}\), used to specify the width of the dead band. The simulations are started at 0 AM on January 1, with the starting conditions that the damper is fully opened (\(s = 90^\circ\)) and the fan is stopped (\(F = 0\) W). In contrary to thermal building simulation programmes, where it is necessary to start the simulations at a previous point in time than the actual simulation start, to account for the history dependent effect of the buildings thermal mass, the simulations can be started immediately.
This is also true when simulating the time dependent stochastic input processes. At the simulation start, the first realisation of the input is obtained according to the marginal distribution and mutual dependence between the input variables. At a later point in time, the input is again generated according to the distribution of the input variables. However, the difference is, that now the realisations of the previous time steps are known and influences the results, due to the time dependency of the load processes. Thus, when multiple simulations are conducted, both the statistical properties for each point in time and the dependence in time will be guarantied.

**Figure 3.7** Damper opening angle for different values of $Q_{fac}$ for a single realisation of the input processes.
Figure 3.8 Fan power supply for different values of $Q_{fac}$ for a single realisation of the input processes.

Figure 3.9 Air flow for different values of $Q_{fac}$ for a single realisation of the input processes.

It can be observed from Figures 3.7 and 3.8, how the control parameters, $s$ and $F$, change during the simulation in order to keep the air flow at the requested 0.3 m$^3$/s. The control parameters can be regarded as a feedback to the system, as they act both as input and output parameters. The air flow
depends on the values of the control parameters, which are adjusted for the next time step. It can also be observed from the figures, that by increasing the width of the dead band \( Q_{\text{fac}} \), the control parameters are constant in some time intervals, such that the air flow can vary freely, resulting in a greater variation of \( Q \).

In order to examine the convergence of the results, the mean values and standard deviation of \( s, F \) and \( Q \) are calculated for 10, 100 and 1000 realisations of the input processes, respectively, using \( Q_{\text{fac}} = 0 \). The results are presented in Figures 3.10 - 3.15.

It is observed that the discrete states where either the fan is off or the damper is opened is not reconcilable in the plots of the mean values and standard deviations for the control parameters, see Figures 3.10 - 3.13. This is due to the discrete nature of the control mechanisms, e.g. in some realisations the fan is stopped where in others the fan is controlled, suggesting that the mean is somewhere between these situations. This also suggests that the problem is not purely described by mean values and standard deviations alone. It can be seen that the level of the mean values and standard deviations is already reached using 100 simulations, eventhough at 1000 simulations, there is still some noise in the results, implying that the results could be refined even better by using more simulations.

![Figure 3.10](image-url)  
**Figure 3.10** Mean value of damper opening angle, \( \mu_s \), for 10, 100 and 1000 input realisations, calculated for \( Q_{\text{fac}} = 0 \).
Figure 3.11  Standard deviation of damper opening angle, $\sigma_s$, for 10, 100 and 1000 input realisations, calculated for $Q_{fac} = 0$.

Figure 3.12  Mean value of fan power, $\mu_F$, supply for 10, 100 and 1000 input realisations, calculated for $Q_{fac} = 0$. 
**Figure 3.13** Standard deviation of fan power supply, $\sigma_F$, for 10, 100 and 1000 input realisations, calculated for $Q_{fac} = 0$.

**Figure 3.14** Mean value of air flow, $\mu_Q$, for 10, 100 and 1000 input realisations, calculated for $Q_{fac} = 0$. 
Figure 3.15  Standard deviation of air flow, $\sigma_Q$, for 10, 100 and 1000 input realisations, calculated for $Q_{fac} = 0$.

To further illustrate this, the distribution functions for $s$, $F$ and $Q$, are estimated by statistical sampling, see Section 2.2, for $t = 12$ h, using 1000 realisations and for different values of $Q_{fac}$. The results are shown in Figures 3.16 - 3.18.

Figure 3.16  Distribution function of damper opening angle, $s$, for $t = 12$ h and for different values of $Q_{fac}$. Based on 1000 simulations.
By looking at the distribution functions for the control parameters, $s$ and $F$, it appears that none of them looks Gaussian distributed. It can be clearly observed from the distribution function of the fan power, that there is a finite probability that the fan is stopped, and a continuous distribution when the fan is running. The same is the case for the distribution of the damper opening angle. There is a discrete probability that the damper is opened and a continuous distribution when the damper is not fully opened. The distribution of the damper opening angle, Figure 3.16, is not influenced as
significantly by the width of the dead band, $Q_{lac}$, as the distribution of the fan power supply, Figure 3.17. The introduction of the dead band implies a more discrete behaviour of the latter distribution function.

The distribution function of the air flow shown in Figure 3.18 behaves continuously. As expected, it can be observed that the distribution is much influenced by the width of the dead band, $Q_{lac}$, a wider band implies a wider range on $Q$.

Considering the control parameters, it is important to notice that a probabilistic approach which is capable of determining only mean values and standard deviation or even continuous distribution functions is insufficient in cases where control strategies are considered, as the discrete nature of the problem must be considered. This may lead to the conclusion that the MCS approach may be the only possibility in such cases, whereas other methods (analytical) may have difficulties dealing with the discrete nature of the problem.

4 Multizone model

In this chapter a simple multizone air flow model is presented. The model is capable of calculating the air flow in a building divided into an arbitrary number of zones and can thus be regarded as a generalisation of the single zone model presented in the previous chapter.

However, at this point, a relatively simple flow equation is implemented in the multizone model to model the different air flow paths. However, in future work it is possible to formulate control strategies, e.g. using dampers and fans as for the single zone model.

The multizone model is coupled with a Monte Carlo Simulation module generating realisations of input data according to the statistical distribution of data. This process is repeated a high number of times, and, finally, statistical treatment of output data results in output distributions and statistics, e.g. mean values and standard deviations. The method is illustrated by a case study.

4.1 Multizone building model

A building is divided into $n$ zones representing rooms or part of rooms, see Figure 4.1. Pressure differences between the zones induce air flow through cracks, openings and ventilation system channels, etc. The main task of a multizone model is to be able to calculate these air flows in order to design the HVAC system and/or natural airing devices, etc.

The main principle of the multizone model is as follows: By demanding that the air mass entering each zone is equal to the air mass exiting, a number of nonlinear air flow equations can be established, one for each zone. Then the nonlinear equation system can be solved by a standard numerical technique for the unknown zone reference pressures. Finally, air flows between the different zones can be calculated.

The pressure differences and thus the air flows between the zones are caused by the following effects, (Haghighat and Rao, 1991):

- Wind induced pressure differences, $\Delta P_W$.
- Stack effects induced by density and height differences, $\Delta P_S$. 
- Pressure differences induced by mechanical ventilation, $\Delta P^M$.

The wind induced pressure difference, $\Delta P^W$, is caused by the wind acting on the building envelope. The stack effect, $\Delta P^S$, is caused by opening height differences and by air density differences between the zones. Finally, pressure differences, $\Delta P^M$, can be enforced by mechanical ventilation devices, i.e. fans etc. The natural driving forces, i.e. $\Delta P^W$ and $\Delta P^S$ are accounted for in the present model, whereas $\Delta P^M$ may be implemented later.

**Figure 4.1** Principle of multizone model. $P$ is local pressure, $Q$ is air flow and $h$ is height.

Each zone is presented by a node with pressure, $P_k(h_k)$, $k = 1, 2, \ldots, n$, situated at the height, $h_k$, above the ground (zero) level, which is used as a reference level for the pressure calculations, see Figure 4.1. The pressure within each zone is assumed hydrostatic distributed according to

$$P_k(h) = P_k(h_k) + \rho_k g (h_k - h) \quad (4.1)$$

where $\rho_k$ is the zone air density, which can be estimated from equation (3.2) as a function of zone air temperature, $\theta_k$, zone water humidity ratio, $W_k$, and atmospheric pressure at ground (zero) level, $P_{atm}$. Both $\theta_k$ and $W_k$ are assumed to be constant within each zone.

Thus, the pressure distribution, $P_k(h)$, within the zone vary linearly with height, $h$, see also Figure 4.2. However, as the multizone model is solved only for zone pressure at reference height, $P_k = P_k(h_k)$, will be used in the following.

As boundary conditions, $m$ nodes with known pressures, $P^0_k$, $k = 1, 2, \ldots, m$, situated at heights, $h^0_k$, above the ground level are introduced. These pressures are used to model the external environment at different heights and locations around the building envelope corresponding to the external openings. The external pressures, $P^0_k$, are depending on the heights of the external nodes/openings, $h^0_k$, through the stack effects, $\Delta P^S,0$, and on the wind pressures on the building envelope, $\Delta P^W,0$, (Haghighat and Rao, 1991), see Figure 4.3.
\[ P_k^0 = P_{atm} + \Delta P_k^S,0 + \Delta P_k^W,0 = P_{atm} - \rho_0 g h_k^0 + \frac{1}{2} \rho_0 C_p v^2 \]  

(4.2)

where:  
\[ \rho_0 = \text{Density of external air (kg/m}^3\) \]  
\[ C_p = \text{Pressure coefficient, related to the geometry of the building (n.d.)} \]  
\[ v = \text{Wind speed corresponding to building height (m/s)} \]

The external air density, \( \rho_0 \) can be obtained from equation (3.2) as a function of the external air temperature, \( \theta_0 \), the water humidity ratio, \( W_0 \), and atmospheric pressure at ground level, \( P_{atm} \). The wind speed, \( v \), is modelled by the same wind profile, as for the single zone model, i.e. equation (3.12).

The air flow between the zones \( k \) and \( l \) with unknown reference pressures, \( P_k \) and \( P_l \), is divided into \( s_{kl} \) components, \( Q_{kl}^c, c = 1, \ldots, s_{kl} \), each modelling a specific type of air flow between the two zones. Similarly, the air flow between the Zone \( k \) with unknown reference pressure and the boundary node \( l \) with known pressure is modelled by an air flow component, \( Q_{kl}^0 \). Thus, one boundary node must be specified for each external opening in the building envelope.

The internal openings corresponding to air flow components, \( Q_{kl}^c \), are situated at heights, \( h_{kl}^c \), and the external openings corresponding to air flow components, \( Q_{kl}^0 \), are situated at heights, \( h_l^0 \). Both types of air flow components are modelled by semi-empirical air flow equations, \( Q_{kl}^c(\Delta P_{kl}^c) \) and \( Q_{kl}^0(\Delta P_{kl}^0) \), relating the pressure differences over the openings, \( \Delta P_{kl}^c \) and \( \Delta P_{kl}^0 \), to the air flows, \( Q_{kl}^c \) and \( Q_{kl}^0 \). In this work, a relatively simple flow equation is used (Feustel and Dieris, 1992), but other types could be considered, e.g. in case of large openings, ductwork, etc.

\[ Q_{kl}^c(\Delta P_{kl}^c) = \begin{cases} 
D_{kl}^c (\Delta P_{kl}^c)^{n_{kl}^c} & \text{if } \Delta P_{kl}^c \geq 0 \\
-D_{kl}^c (-\Delta P_{kl}^c)^{n_{kl}^c} & \text{else} 
\end{cases} 
\]  

(4.3)

where \( D_{kl}^c \) and \( n_{kl}^c \) are empirical coefficients. \( D_{kl}^c \) varies over a wide range, depending on opening type and size, the exponent \( n_{kl}^c \) is typical varying from 0.50 – 0.85, see Orme et. al, 1998.

Pressure differences, \( \Delta P_{kl}^M \), induced by mechanical ventilation devices can be accounted for by specifying an air flow equation corresponding to the specific device. However, this is not considered in this work.

As the zone pressure distribution is assumed hydrostatic, the zone pressures at the opening heights, \( h_{kl}^c \) or \( h_l^0 \), are different than the reference zone pressures at heights, \( h_k \) and \( h_l \), which is also illustrated in Figures 4.2 and 4.3.
The pressure difference, $\Delta P_{kl}^c$, over an internal opening corresponding to the air flow component, $Q_{kl}^c$, is given by

$$\Delta P_{kl}^c(P_k, P_l) = P_{kl}^c(P_k) - P_{kl}^c(P_l)$$

$$= p_k + \Delta P_{k,kl}^c - p_l - \Delta P_{l,kl}^c$$

$$= p_k - p_l + \rho_k g h_k - p_l - \rho_l g h_l^c$$

where $P_{kl}^c$ and $P_{kl}^c$ are the zone pressures in Zones $k$ and $l$ respectively at opening height, $h_k^c$, and where $\Delta P_{k,kl}^c$ and $\Delta P_{l,kl}^c$ are the corrections to the reference zone pressures to adjust for the opening height. These adjustments are also termed stack effects (therefore the superscript $S$) accounting for the importance of opening height and density differences between the two zones.

Similarly, as illustrated in Figure 4.3, the pressure difference, $\Delta P_{kl}^0$, over an external opening corresponding to the air flow component, $Q_{kl}^0$, is given by

$$\Delta P_{kl}^0(P_k) = P_{kl}^0(P_k) - P_{kl}^0(P_l)$$

$$= p_k + \Delta P_{k,kl}^0 - p_l$$

$$= p_k - p_l + \rho_k g (h_k - h_l^0) - p_l$$

Figure 4.2 Pressure difference between points on opposite sides of internal opening.

Figure 4.3 Pressure difference between points on opposite sides of external opening.
For each internal zone the air flow into the zone must balance the air flow out of the zone in order to satisfy the mass balance equation. The various air flow components corresponding to a Zone $i$ are illustrated in Figure 4.4.

![Figure 4.4 Example of air flow components corresponding to Zone i.](image)

By defining air flow out of the zone to be positive, the mass balance equation for Zone $i$ can be written as

$$g_i = \sum_{k=1}^{a} \sum_{i=1}^{n} g_{i,k}^{\varepsilon} + \sum_{k=1}^{n} \sum_{l=1}^{m} g_{i,l}^{0} = 0 \quad i = 1, \ldots, n$$

where

$$g_{i,k}^{\varepsilon} = \begin{cases} Q_{ki}^{\varepsilon} \left( \Delta P_{ki}^{\varepsilon}(P_k, P_i) \right) & \text{if } i = k \\ -Q_{ki}^{\varepsilon} \left( \Delta P_{ki}^{\varepsilon}(P_k, P_i) \right) & \text{if } i = l \\ 0 & \text{else} \end{cases}$$

$$g_{i,l}^{0} = \begin{cases} Q_{il}^{0} \left( \Delta P_{il}^{0}(P_i, P_l) \right) & \text{if } i = k \\ 0 & \text{else} \end{cases}$$

In the actual implementation computer implementation of equation (4.6), looping is done over all existing air flow components instead of the nodes, as many nodes will not be connected to each other by air flow equations.

In the single zone case, see Chapter 3, it is possible to solve the single nonlinear equation for the unknown air flow directly. However, this is not the case in the multizone case, as multiple air flow paths may exist between the zones, and between the zones and the external air. Therefore, the air flow equation system is solved for the zone reference pressures instead, where after the air flows can be calculated.

Thus, one mass balance equation is derived for each zone with unknown pressure and a nonlinear equation system with $n$ equations and $n$ unknown pressures is assembled. The equation system is solved for the unknown pressure vector, $P$, by the solution algorithm described in the next section.

### 4.2 Solution of the air flow equations

The flow equation system is solved by Newton’s method, which is merely a generalisation of the approach used in the single zone case, see section 3.6. However, in the multizone case, a nonlinear
The equation system, equation (4.6), is rewritten as

\[ g(P) = 0 \]  \hspace{1cm} (4.9)

where \( P = [P_1 \ldots P_n]^T \) is the vector of unknown pressures.

By Taylor series expansion of equation (4.9) based on the vector, \( P_0 \), one obtains

\[ g(P) \approx g(P_0) + \left. \frac{dg}{dP} \right|_{P_0} (P - P_0) \]  \hspace{1cm} (4.10)

Equation (4.10) is solved for the difference between the pressures

\[ dP = P - P_0 = \left( \left. \frac{dg}{dP} \right|_{P_0} \right)^{-1} g(P_0) \]  \hspace{1cm} (4.11)

If \( P_0 \) is chosen as a guess of the unknown pressure vector, an updated pressure vector, \( P \), can be obtained by

\[ P = P_0 + dP \]  \hspace{1cm} (4.12)

The standard Newton’s method consists of iterating in equations (4.11) and (4.12), until a converged pressure vector is obtained. After iteration, \( P_0 \) is replaced with the updated pressure vector, \( P \), before the next iteration is conducted.

However, numerical tests of the standard Newton’s method indicate that occasional instances of very slow convergence occur, with oscillation corrections, \( dP \), on successive iterations, see Walton (1995). This can be partly avoided by using the following Steffensen iteration scheme

\[ P_i = P_{0,i} + \frac{dP_i}{1 - r_i} \]  \hspace{1cm} (4.13)

where \( r_i \) is the ratio of \( dP_i \) for the current iteration to its value for the previous iteration. Using Steffensen iteration, equation (4.12) is replaced with equation (4.13) during the iteration process, and it can be observed that if \( r_i = 0, i = 1, 2, \ldots, n \), equation (4.13) is identical to equation (4.12).

In order to obtain the derivative \( dg/dP \) in equation (4.11), which is denoted the Jacobian matrix, differentiation of equation (4.6) is conducted

\[ \frac{dg_i}{dP_j} = \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{c=1}^{s_{ij}} \frac{dg_{i,kl}}{dP_j} + \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{dg_{i,kl}^0}{dP_j} \]

\[ = \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{c=1}^{s_{ij}} \frac{dg_{i,kl}^c}{dQ_{kl}^c} \frac{dQ_{kl}^c}{dP_j} \frac{dP_j}{dP_j} + \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{dg_{i,kl}^0}{dQ_{kl}^0} \frac{dQ_{kl}^0}{dP_j} \frac{dP_j}{dP_j} \]  \hspace{1cm} (4.14)

\[ i, j = 1, \ldots, n \]
where it has been used that $g^c_{i,kl} = g^c_{i,kl} \left( Q^c_{kl} (\Delta P^c_{kl}) \right)$ and that $g^0_{i,kl} = g^0_{i,kl} \left( Q^0_{kl} (\Delta P^0_{kl}) \right)$.

The derivative of the flow equation (4.3) is

$$\frac{dQ^c_{kl}}{dP^c_{kl}} = \begin{cases} n^c_{kl} D^c_{kl} \left[ P^c_{kl} \right]^{-1} & \text{if } \Delta P^c_{kl} \neq 0 \\ \text{Not defined} & \text{else} \end{cases} \quad (4.15)$$

A similar expression is obtained for $dQ^0_{kl} / d\Delta P^0_{kl}$.

The pressure derivatives of $\Delta P^c_{kl}$ are obtained by differentiation of equation (4.4)

$$\frac{d\Delta P^c_{kl}}{dP_j} = \begin{cases} 1 & \text{if } j = k \\ -1 & \text{if } j = l \\ 0 & \text{else} \end{cases} \quad (4.16)$$

Finally, the pressure derivative of $\Delta P^0_{kl}$ is obtained from equation (4.5)

$$\frac{d\Delta P^0_{kl}}{dP_j} = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{else} \end{cases} \quad (4.17)$$

When the unknown pressure vector, $P$, has been obtained, the various flow components, $Q^c_{kl}$ and $Q^0_{kl}$, are calculated using the flow equations, see Chapter 4.1.

### 4.3 Test case

The multizone building shown in Figure 4.5 is selected for analysis. The building consists of four floors and 12 zones. It is orientated on an East-West axis, with zones 1, 4, 7 and 10 situated on the West side, whereas zones 3, 6, 9 and 12 are situated at the East side. The openings are fixed in time (no control is assumed), and therefore only a single point in time is analysed. The simulation is performed corresponding to the situation at 12 AM on January 1 in the Danish DRY. Figure 4.6 shows the corresponding definitions of pressure nodes and air flow paths.
Figure 4.5 Geometry of test case. A, B and C denote opening types as specified in Table 4.1.

Figure 4.6 Definition of air flow paths and zone reference pressures.

Three opening types are defined, type A, B and C, as shown in Figure 4.5 and with air flow parameters given in Table 4.1. Table 4.2 lists the room air temperature and the water humidity ratios of the zones and in Table 4.3 the deterministic parameters defining the external climate are shown.

Table 4.1 Opening types as referred to Figure 4.5.

<table>
<thead>
<tr>
<th>Opening types</th>
<th>$D$ (m$^3$/s)/(Pa)</th>
<th>$n$ (n.d.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.04</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>0.02</td>
<td>0.6</td>
</tr>
<tr>
<td>C</td>
<td>0.005</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Table 4.2 Deterministic zone parameters.

<table>
<thead>
<tr>
<th>Zones</th>
<th>$\theta$ (deg. C)</th>
<th>$W$ (kg/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,3</td>
<td>20</td>
<td>0.008</td>
</tr>
<tr>
<td>4,5,6</td>
<td>21</td>
<td>0.008</td>
</tr>
<tr>
<td>7,8,9</td>
<td>22</td>
<td>0.008</td>
</tr>
<tr>
<td>10,11,12</td>
<td>23</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 4.3 Deterministic external climate parameters.

<table>
<thead>
<tr>
<th>$P_{atm}$ (Pa)</th>
<th>$W_0$ (kg/kg)</th>
<th>$z_0$ (m)</th>
<th>$H$ (m)</th>
<th>$\alpha$ (n.d.)</th>
<th>$g$ (m/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101325</td>
<td>0.006</td>
<td>10</td>
<td>12</td>
<td>0.28</td>
<td>9.82</td>
</tr>
</tbody>
</table>

The pressure coefficients ($C_p$-values) for the external openings are adapted from Orme et al. 1998, where pressure coefficients are listed for rectangular building as a function of wind direction and the relative position on the surfaces of the building. The values used for the simulation are given in Figures 4.7 - 4.9 for each.

Figure 4.7 Pressure coefficients for West openings.

Figure 4.8 Pressure coefficients for East openings.
The external air temperature, the wind speed and direction are simulated by the same probabilistic model as used for the single zone model in Chapter 3. However, the only difference here is that only a single time step is considered.

Tables 4.4 and 4.5 show the resulting mean values and standard deviations for each of the air flow components based on 10, 100, 1000 and 10000 simulations, respectively.
### Table 4.4 Mean values of air flow components.

<table>
<thead>
<tr>
<th>Flow Component</th>
<th>10 Realisations</th>
<th>100 Realisations</th>
<th>1000 Realisations</th>
<th>10000 Realisations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{Q_{1,2}} ) (m³/s)</td>
<td>0.0475</td>
<td>0.0433</td>
<td>0.0412</td>
<td>0.0389</td>
</tr>
<tr>
<td>( \mu_{Q_{2,3}} ) (m³/s)</td>
<td>0.0030</td>
<td>0.0035</td>
<td>0.0016</td>
<td>-0.0006</td>
</tr>
<tr>
<td>( \mu_{Q_{2,4}} ) (m³/s)</td>
<td>0.0320</td>
<td>0.0330</td>
<td>0.0316</td>
<td>0.0295</td>
</tr>
<tr>
<td>( \mu_{Q_{3,5}} ) (m³/s)</td>
<td>0.0164</td>
<td>0.0148</td>
<td>0.0127</td>
<td>0.0106</td>
</tr>
<tr>
<td>( \mu_{Q_{3,6}} ) (m³/s)</td>
<td>0.0316</td>
<td>0.0324</td>
<td>0.0307</td>
<td>0.0285</td>
</tr>
<tr>
<td>( \mu_{Q_{3,8}} ) (m³/s)</td>
<td>0.0184</td>
<td>0.0182</td>
<td>0.0160</td>
<td>0.0140</td>
</tr>
<tr>
<td>( \mu_{Q_{3,9}} ) (m³/s)</td>
<td>0.0238</td>
<td>0.0243</td>
<td>0.0226</td>
<td>0.0204</td>
</tr>
<tr>
<td>( \mu_{Q_{4,1}} ) (m³/s)</td>
<td>0.0232</td>
<td>0.0232</td>
<td>0.0213</td>
<td>0.0194</td>
</tr>
<tr>
<td>( \mu_{Q_{4,2}} ) (m³/s)</td>
<td>0.0445</td>
<td>0.0398</td>
<td>0.0396</td>
<td>0.0394</td>
</tr>
<tr>
<td>( \mu_{Q_{4,4}} ) (m³/s)</td>
<td>0.0601</td>
<td>0.0580</td>
<td>0.0585</td>
<td>0.0583</td>
</tr>
<tr>
<td>( \mu_{Q_{4,5}} ) (m³/s)</td>
<td>0.0733</td>
<td>0.0722</td>
<td>0.0732</td>
<td>0.0727</td>
</tr>
<tr>
<td>( \mu_{Q_{4,6}} ) (m³/s)</td>
<td>0.0065</td>
<td>0.0054</td>
<td>0.0053</td>
<td>0.0053</td>
</tr>
<tr>
<td>( \mu_{Q_{4,7}} ) (m³/s)</td>
<td>0.0065</td>
<td>0.0059</td>
<td>0.0058</td>
<td>0.0058</td>
</tr>
<tr>
<td>( \mu_{Q_{4,8}} ) (m³/s)</td>
<td>0.0068</td>
<td>0.0065</td>
<td>0.0066</td>
<td>0.0066</td>
</tr>
<tr>
<td>( \mu_{Q_{4,9}} ) (m³/s)</td>
<td>0.0071</td>
<td>0.0072</td>
<td>0.0072</td>
<td>0.0072</td>
</tr>
<tr>
<td>( \mu_{Q_{4,10}} ) (m³/s)</td>
<td>0.0097</td>
<td>0.0096</td>
<td>0.0097</td>
<td>0.0096</td>
</tr>
<tr>
<td>( \mu_{Q_{4,12}} ) (m³/s)</td>
<td>0.0094</td>
<td>0.0092</td>
<td>0.0093</td>
<td>0.0093</td>
</tr>
<tr>
<td>( \mu_{Q_{5,1}} ) (m³/s)</td>
<td>-0.0540</td>
<td>-0.0487</td>
<td>-0.0465</td>
<td>-0.0442</td>
</tr>
<tr>
<td>( \mu_{Q_{5,2}} ) (m³/s)</td>
<td>-0.0034</td>
<td>-0.0024</td>
<td>-0.0042</td>
<td>-0.0064</td>
</tr>
<tr>
<td>( \mu_{Q_{5,3}} ) (m³/s)</td>
<td>-0.0323</td>
<td>-0.0341</td>
<td>-0.0329</td>
<td>-0.0307</td>
</tr>
<tr>
<td>( \mu_{Q_{5,4}} ) (m³/s)</td>
<td>0.0158</td>
<td>0.0136</td>
<td>0.0113</td>
<td>0.0093</td>
</tr>
<tr>
<td>( \mu_{Q_{5,5}} ) (m³/s)</td>
<td>-0.0345</td>
<td>-0.0355</td>
<td>-0.0338</td>
<td>-0.0315</td>
</tr>
<tr>
<td>( \mu_{Q_{5,6}} ) (m³/s)</td>
<td>0.0162</td>
<td>0.0161</td>
<td>0.0139</td>
<td>0.0119</td>
</tr>
<tr>
<td>( \mu_{Q_{5,7}} ) (m³/s)</td>
<td>-0.0363</td>
<td>-0.0370</td>
<td>-0.0353</td>
<td>-0.0328</td>
</tr>
<tr>
<td>( \mu_{Q_{5,8}} ) (m³/s)</td>
<td>0.0151</td>
<td>0.0163</td>
<td>0.0142</td>
<td>0.0123</td>
</tr>
<tr>
<td>( \mu_{Q_{5,9}} ) (m³/s)</td>
<td>0.0221</td>
<td>0.0223</td>
<td>0.0224</td>
<td>0.0220</td>
</tr>
<tr>
<td>( \mu_{Q_{5,10}} ) (m³/s)</td>
<td>0.0739</td>
<td>0.0733</td>
<td>0.0744</td>
<td>0.0737</td>
</tr>
<tr>
<td>( \mu_{Q_{5,11}} ) (m³/s)</td>
<td>0.0174</td>
<td>0.0161</td>
<td>0.0164</td>
<td>0.0164</td>
</tr>
</tbody>
</table>
Table 4.5 Standard deviations of air flow components.

<table>
<thead>
<tr>
<th>Flow Component</th>
<th>10 Realisations</th>
<th>100 Realisations</th>
<th>1000 Realisations</th>
<th>10000 Realisations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{q_{1,1}}$ (m$^3$/s)</td>
<td>0.0466</td>
<td>0.0432</td>
<td>0.0458</td>
<td>0.0459</td>
</tr>
<tr>
<td>$\sigma_{q_{2,3}}$ (m$^3$/s)</td>
<td>0.0558</td>
<td>0.0481</td>
<td>0.0495</td>
<td>0.0490</td>
</tr>
<tr>
<td>$\sigma_{q_{3,4}}$ (m$^3$/s)</td>
<td>0.0542</td>
<td>0.0500</td>
<td>0.0522</td>
<td>0.0519</td>
</tr>
<tr>
<td>$\sigma_{q_{4,4}}$ (m$^3$/s)</td>
<td>0.0517</td>
<td>0.0462</td>
<td>0.0483</td>
<td>0.0481</td>
</tr>
<tr>
<td>$\sigma_{q_{5,5}}$ (m$^3$/s)</td>
<td>0.0622</td>
<td>0.0583</td>
<td>0.0606</td>
<td>0.0598</td>
</tr>
<tr>
<td>$\sigma_{q_{6,6}}$ (m$^3$/s)</td>
<td>0.0497</td>
<td>0.0450</td>
<td>0.0474</td>
<td>0.0478</td>
</tr>
<tr>
<td>$\sigma_{q_{7,7}}$ (m$^3$/s)</td>
<td>0.0621</td>
<td>0.0584</td>
<td>0.0604</td>
<td>0.0595</td>
</tr>
<tr>
<td>$\sigma_{q_{8,8}}$ (m$^3$/s)</td>
<td>0.0456</td>
<td>0.0399</td>
<td>0.0422</td>
<td>0.0429</td>
</tr>
<tr>
<td>$\sigma_{q_{9,9}}$ (m$^3$/s)</td>
<td>0.0223</td>
<td>0.0138</td>
<td>0.0124</td>
<td>0.0119</td>
</tr>
<tr>
<td>$\sigma_{q_{10,10}}$ (m$^3$/s)</td>
<td>0.0162</td>
<td>0.0134</td>
<td>0.0131</td>
<td>0.0128</td>
</tr>
<tr>
<td>$\sigma_{q_{11,11}}$ (m$^3$/s)</td>
<td>0.0193</td>
<td>0.0231</td>
<td>0.0243</td>
<td>0.0237</td>
</tr>
<tr>
<td>$\sigma_{q_{12,12}}$ (m$^3$/s)</td>
<td>0.0034</td>
<td>0.0027</td>
<td>0.0025</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\sigma_{q_{13,13}}$ (m$^3$/s)</td>
<td>0.0023</td>
<td>0.0020</td>
<td>0.0018</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\sigma_{q_{14,14}}$ (m$^3$/s)</td>
<td>0.0020</td>
<td>0.0023</td>
<td>0.0023</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\sigma_{q_{15,15}}$ (m$^3$/s)</td>
<td>0.027</td>
<td>0.021</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>$\sigma_{q_{16,16}}$ (m$^3$/s)</td>
<td>0.0026</td>
<td>0.0031</td>
<td>0.0032</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\sigma_{q_{17,17}}$ (m$^3$/s)</td>
<td>0.0020</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\sigma_{q_{18,18}}$ (m$^3$/s)</td>
<td>0.0459</td>
<td>0.0419</td>
<td>0.0447</td>
<td>0.0448</td>
</tr>
<tr>
<td>$\sigma_{q_{19,19}}$ (m$^3$/s)</td>
<td>0.0574</td>
<td>0.0488</td>
<td>0.0500</td>
<td>0.0494</td>
</tr>
<tr>
<td>$\sigma_{q_{20,20}}$ (m$^3$/s)</td>
<td>0.0538</td>
<td>0.0503</td>
<td>0.0523</td>
<td>0.0520</td>
</tr>
<tr>
<td>$\sigma_{q_{21,21}}$ (m$^3$/s)</td>
<td>0.0522</td>
<td>0.0461</td>
<td>0.0483</td>
<td>0.0481</td>
</tr>
<tr>
<td>$\sigma_{q_{22,22}}$ (m$^3$/s)</td>
<td>0.0659</td>
<td>0.0620</td>
<td>0.0644</td>
<td>0.0634</td>
</tr>
<tr>
<td>$\sigma_{q_{23,23}}$ (m$^3$/s)</td>
<td>0.0487</td>
<td>0.0442</td>
<td>0.0466</td>
<td>0.0472</td>
</tr>
<tr>
<td>$\sigma_{q_{24,24}}$ (m$^3$/s)</td>
<td>0.0772</td>
<td>0.0732</td>
<td>0.0757</td>
<td>0.0741</td>
</tr>
<tr>
<td>$\sigma_{q_{25,25}}$ (m$^3$/s)</td>
<td>0.0468</td>
<td>0.0414</td>
<td>0.0440</td>
<td>0.0454</td>
</tr>
<tr>
<td>$\sigma_{q_{26,26}}$ (m$^3$/s)</td>
<td>0.0177</td>
<td>0.0182</td>
<td>0.0189</td>
<td>0.0183</td>
</tr>
<tr>
<td>$\sigma_{q_{27,27}}$ (m$^3$/s)</td>
<td>0.0441</td>
<td>0.0492</td>
<td>0.0512</td>
<td>0.0500</td>
</tr>
<tr>
<td>$\sigma_{q_{28,28}}$ (m$^3$/s)</td>
<td>0.0089</td>
<td>0.0086</td>
<td>0.0094</td>
<td>0.0095</td>
</tr>
</tbody>
</table>

It can be observed, that both mean values and standard deviations are not fully converged at 10,000 simulations, however sufficient precise results are obtained even after 100 simulations. To further investigate the results, the empirical distribution functions of the air flow components, $Q_{2,3}, Q_{8,11}$ and $Q_{4,2}$ are calculated by equation (2.14) and are plotted in Figures 4.10-4.12.
Figure 4.10  Empirical distribution function of air flow component, $Q_{2,3}$ obtained by 10, 100, 1000 and 10000 simulations respectively.

Figure 4.11  Empirical distribution function of air flow component, $Q_{8,11}$ obtained by 10, 100, 1000 and 10000 simulations respectively.
Figure 4.12 Empirical distribution function of air flow component, $Q_{4,2}^0$, obtained by 10, 100, 1000 and 10000 simulations, respectively.

By inspection of the figures, it can be seen that the empirical distribution functions are almost converged when using 10000 simulations. The air flow component, $Q_{2,3}$, corresponds to a horizontal internal opening and the empirical distribution function in Figure 4.10 shows that both negative and positive air flows are present during the simulations. The air flow component, $Q_{4,11}$, corresponds to a vertical internal opening. Now, only positive air flows are present, see Figure 4.11. Even though the wind is blowing from all directions, the stack effect will ensure that the air is flowing upwards within the building. Finally, air flow component, $Q_{0,2}$, corresponds to a horizontal opening in the building façade. The same overall behaviour as in the case of the internal horizontal opening case, $Q_{2,3}$, see Figure 4.12.

By looking at the empirical distribution plots and the mean air flows in Table 4.4, it can be concluded that components corresponding to horizontal air flow, nearly all have a mean value of approximate zero, and are distributed with flow in both directions. This is caused partly by the symmetry of the building, and partly because no control is enforced to keep a specified air flow, as was the case for the single zone model of Chapter 3. The components corresponding to vertical air flow, are distributed such that the air always flows upwards. This is due to the stack effect and the pressure distribution over the building envelope. Even though no control is considered, it is observed that various air flow components are non-Gaussian distributed, due to the nonlinear nature of the air flow problem. Thus MCS may be the only possibility of stochastic multizone analysis.

Next step will be to include control strategies like in the single zone case. In order to do so, more air flow components must be formulated, depending on various control parameters, i.e. dampers, fans etc. The damper and fan described in chapter 3 cannot be implemented directly, as the air flow equation in that case where based on the air flow as an unknown, whereas in the multizone case, the unknown zone pressures are used instead.
5 Conclusion

In this report, a stochastic single zone air flow model for a hybrid ventilated building is presented. Natural driving forces from wind, height and air temperature differences are used to drive air through the enclosure. The air flow is controlled by a damper and a fan. The control of the damper and the fan is performed on basis of the air flow at a previous time, thus, a feedback loop is present in this model.

The Monte Carlo Simulation (MCS) approach is used to estimate the statistics of the resulting air flow and control parameters, i.e. mean values, standard deviations and distributions functions. Thus, one is able to more realistically examine the behaviour of a ventilation system, taking into account the variability and statistical properties of the external weather climate. In a conventional deterministic case, where the input consists of short representative time series of the input loads, important statistical properties is lacking.

Using the MCS method, multiple realisations of input time series are generated from the input statistics using simulation techniques. Probabilistic models for the input, i.e. external air temperature, wind speed and directions, are adapted from the Danish DRY. For each input time series, a corresponding time series of the output is generated, and the output statistics and distribution functions can then be estimated by statistical sampling.

A multizone model is presented, which can be regarded as a generalisation of the single zone model. The model is capable of predicting air flow and pressure distributions within a building that is divided into an arbitrary number of zones and air flow paths. The air flows are driven by pressure differences due to wind and stack pressure differences. The MCS approach is again used to obtain the statistics of the resulting air flows, i.e. mean values, standard deviations and distribution functions.

Future work on the multizone model may focus on implementing control strategies. By considering the multizone program as a “black box” in the MCS module, it will be possible to use the stochastic approach with commercial multizone programmes. By using a “black box” approach, it is also believed that the computer codes will be of practical value for the HVAC system designer as he can use the codes without in depth knowledge of the underlying theories.

Another interesting further development will be to couple probabilistic multi (or single) zone air flow models with probabilistic thermal simulation models. In reality, the air flows are functions of the air temperatures (the zone temperatures are regarded as specified input in the multizone model), which again are depending on the air flows, transferring heat between various parts of the building and between the building and the external environment. Thus, a coupled formulation is required in order to solve for both air flows and temperature distribution.

In the single/multizone case, a mass balance is formulated, resulting in a nonlinear equation system to be solved. However, in the thermal building simulation case, a heat balance is formulated, and due to the thermal capacity of the building, a differential equation system is to be solved. In Brohus et al. 2002b, Stochastic Differential Equations (SDE) are used to formulate a probabilistic thermal building simulation model, and a simple model for natural ventilation is proposed. However, a mayor step forward would be to be to consider a general coupled probabilistic air flow and thermal simulation program.
Acknowledgements

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Appendix A

Alternative derivation of Single zone model

In Chapter 3, the air flow equation for the single zone model was derived using the air flow, \( Q \), as an unknown. Alternatively, it is possible to express the air flow equation based on the zone reference pressure, \( P \), similar to the multizone model.

The mass air flow through the damper, \( Q_{damp} \), must balance the air flow through the fan, \( Q_{mech} \). By defining that air flow into the front opening (damper) is positive and that air flow out of the top opening (fan) is positive, the mass conservation principle gives

\[
Q_{damp} = Q_{mech}
\]  
(A.1)

The air flow through the damper is related to the damper pressure difference, \( \Delta P_{damp} \), and the opening angle, \( s \), by (3.37)

\[
\Delta P_{damp}(Q_{damp}, s) = R(s)Q_{damp}^2
\]  
(A.2)

which gives

\[
Q_{damp}(\Delta P_{damp}, s) = \begin{cases} \sqrt{\frac{\Delta P_{damp}}{R(s)}} & \text{for } \Delta P_{damp} \geq 0 \\ -\sqrt{\frac{-\Delta P_{damp}}{R(s)}} & \text{else} \end{cases}
\]  
(A.3)

Similarly, the air flow through the fan is related to the fan pressure difference, \( \Delta P_{mech} \), the power supply, \( F \), the internal room air temperature, \( \theta_{\text{int}} \), and the internal water humidity ratio, \( W_{\text{int}} \), by the analytical model of Section 3.4, which in general terms is stated by

\[
\Delta P_{mech} = f(Q_{mech}, F, \theta_{\text{int}}, W_{\text{int}})
\]  
(A.4)

Equation (A.4) is nonlinear and must be solved iteratively by Newton’s method for the air flow. By considering \( F, \theta_{\text{int}} \) and \( W_{\text{int}} \) as constants the solution is written

\[
Q_{mech} = f^{-1}(\Delta P_{mech}, F, \theta_{\text{int}}, W_{\text{int}})
\]  
(A.5)

The pressure difference over the damper and the fan are given by (3.6) and (3.7) respectively.

\[
\Delta P_{damp} = P_{\text{ext}}^{\text{front}} - P_{\text{int}}^{\text{front}} = P_{\text{atm}} - P + \Delta P_{\text{wind}} + (\rho_{\text{int}} - \rho_{\text{ext}})gh_1
\]  
(A.6)

\[
\Delta P_{mech} = P_{\text{ext}}^{\text{top}} - P_{\text{int}}^{\text{top}} = P_{\text{atm}} - P + \Delta P_{\text{wind}} + (\rho_{\text{int}} - \rho_{\text{ext}})gh_2
\]  
(A.7)

By inserting (A.6), (A.7), (A.3) and (A.5) into (A.1), the following nonlinear equation in \( P \) is obtained.
Equation (A.8) is solved iteratively by Newton’s method for the zone reference pressure, $P$, after which the air flow, $Q = Q_{\text{damp}}$, can be obtained by (A.3).

Solving equation (A.8) for the pressure, $P$, is identical to solving equation (3.8) for the air flow, $Q$, due to the fact that the same underlying assumptions are used, i.e. hydrostatic zone pressure distribution and mass conservation. However, in the case of a multizone model where multiple air flow paths exists, it is only possible to formulate an equation system based on the zone reference pressures, $P$, as an air flow based equation system would be over determinate (i.e. more unknowns as equations). The reason to choose the air flow based model for the single zone model was simply to avoid the numerical solution for the fan model, equation (A.5).