WP-1 Hydrodynamics: Background and Strategy

Harry B. Bingham
Robert Read (Post-doc)
Torben Christiansen (PhD)

Eric D. Christensen
Hans Hansen

Mech. Engineering
Tech. Univ. of Denmark
Lygnby, Denmark

DHI Water and Environment
Hørsholm, Denmark

SDWED 1st Symposium, DHI, Hørsholm, Denmark
August 30-31, 2010
Schematic of the hydrodynamic problem

- Deep water waves
- Wave forcing at the device
- Refraction from the bottom
- Triad interaction
- Diffraction from coastal boundaries
- Wave farm
$s > 100\text{m}$

$10 - 50\text{m}$
WP-1 Scope

1. Predicting the *Incident Wave* climate at the device location (10 - 50m water depth)
   - Wave refraction and diffraction plus nonlinearity are all important
   - No simple description of the waves exists
   - Efficient potential flow solvers can do the job (OceanWave3D - DTU)

2. Mildly-nonlinear wave-structure interaction - Optimization and long-term loading
   - Linear hydrodynamic coefficients
   - Nonlinear hydrostatics, mooring system, and PTO forces included in the equations of motion (WAMSIM - DHI/DTU)
   - Model under development to compute linear hydrodynamics on a variable depth, possibly semi-enclosed domain (Rob Read - DTU)

3. Strongly nonlinear wave-structure interaction - Final design analysis
   - Fully nonlinear inviscid flow solver (PhD - DTU)
   - Viscous flow solver (DHI)
The incident waves - OceanWave3D

- 3D potential flow
- Solve using high-order finite-differences on overlapping, boundary-fitted blocks
- Numerical order of accuracy is flexible
- Two paths to convergence $h$-type and $p$-type
The nonlinear potential flow wave problem

\[
\begin{align*}
\zeta(x, t) &= -\nabla \zeta \cdot \nabla \tilde{\phi} + \tilde{w}(1 + \nabla \zeta \cdot \nabla \zeta) \\
\tilde{\phi}_t &= -g \zeta - \frac{1}{2} \nabla \tilde{\phi} \cdot \nabla \tilde{\phi} + \frac{1}{2} \tilde{w}^2 (1 + \nabla \zeta \cdot \nabla \zeta)
\end{align*}
\]
Laplace Problem for $\tilde{w}^1$

$$\nabla^2 \phi + \phi_{zz} = 0, \quad -h < z < \zeta$$

$$\phi_z + \nabla h \cdot \nabla \phi = 0, \quad z = -h$$

---

Laplace Problem for $\tilde{w}$

\[
\nabla^2 \phi + \phi_{zz} = 0, \quad -h < z < \zeta \\
\phi_z + \nabla h \cdot \nabla \phi = 0, \quad z = -h
\]

Sigma transform the vertical coordinate:

\[
\sigma(x, z, t) = \frac{z + h(x)}{\zeta(x, t) + h(x)}
\]

---

Laplace Problem for $\tilde{w}$ \(^1\)

\[
\nabla^2 \phi + \phi_{zz} = 0, \quad -h < z < \zeta
\]

\[
\phi_z + \nabla h \cdot \nabla \phi = 0, \quad z = -h
\]

Sigma transform the vertical coordinate:

\[
\sigma(x, z, t) = \frac{z + h(x)}{\zeta(x, t) + h(x)}
\]

\[
\Phi = \tilde{\phi}, \quad \sigma = 1
\]

\[
\nabla^2 \Phi + \nabla^2 \sigma (\partial_{\sigma} \Phi) + 2 \nabla \sigma \cdot \nabla (\partial_{\sigma} \Phi) + \left( \nabla \sigma \cdot \nabla + \sigma_z^2 \right) (\partial_{\sigma \sigma} \Phi) = 0, \quad 0 \leq \sigma < 1
\]

\[
(\partial_z \sigma + \nabla h \cdot \nabla \sigma) (\partial_{\sigma} \Phi) + \nabla h \cdot \nabla \Phi = 0, \quad \sigma = 0
\]

with $\Phi(x, \sigma, t) = \phi(x, z, t)$

- Gives a fixed computational geometry, no need to re-grid

---

Solution by Arbitrary-Order Finite Differences

- Structured, but non-uniform grid.
- Choose $r$ neighbors to develop 1D, $r - 1$ order FD schemes.
  - Leads to a linear system
    \[ A\mathbf{x} = \mathbf{b} \]
    \( A \) is sparse with at most $r^d$, non-zeros, in $d = 2, 3$ dimensions.
- GMRES iterative solution preconditioned by the linearized, 2nd-order version of the matrix $A_2$:
  \[ A_2^{-1} \{ A(t) \mathbf{x} = \mathbf{b} \} \]
- Multigrid solution of the preconditioning step.
- Solution in $O(10)$ iterations, independent of physics and $N$.
- Time stepping by the classical 4th-order Runge-Kutta scheme.

\[ ^2 \text{Engsig-Karup, Bingham & Lindberg (2009) J. Comp. Phys. 228} \]
Solution by Arbitrary-Order Finite Differences

- Structured, but non-uniform grid.
- Choose $r$ neighbors to develop 1D, $r-1$ order FD schemes.
- Leads to a linear system
  \[ Ax = b \]
  
  $A$ is sparse with at most $r^d$, non-zeros, in $d = 2, 3$ dimensions.
- GMRES iterative solution preconditioned by the linearized, 2nd-order version of the matrix $A_2$:
  \[ A_2^{-1} \{ A(t) x = b \} \]
- Multigrid solution of the preconditioning step.
- Solution in $O(10)$ iterations, independent of physics and $N$.
- Time stepping by the classical 4th-order Runge-Kutta scheme.

---

Scaling of the solution in 3D
Nonlinear test case, 6th-order accurate operators

CPU time

RAM memory use
Error in linear dispersion at different resolutions

- **r=3 uniform grid, \(N_x=50\)**

- **r=3 uniform grid, \(N_x=150\)**

- **r=5 clustered grid, \(N_x=15\)**

- **r=7 clustered grid, \(N_x=9\)**
Highly nonlinear waves in 3D, $kh = 2\pi$, $H/L = 90\%$

30 x 7.5 wavelengths, $N = 1025 \times 257 \times 10 = 2.6 \times 10^6$, $C_r = 0.5$. (6th-order. Approx. 5 min. CPU/wave period.)
Extensions in progress

- Parallelization of the code
- Introduction of a wave-breaking model
- Introduction of a shoreline run-up model
Assumptions

- Small wave steepness at the structure (linear hydrodynamics).
- Mooring/PTO adds: Nonlinearity, and resonant horizontal modes (models provided by other WPs).
- Body motions are still small.

Incident wave and motions problems are de-coupled.

1. Wave forcing: OceanWave3D + Haskind relations.
2. Wave-body interaction: Panel Method (WAMIT), OceanWave3D (under development)

---

\[ H.B. \text{ Bingham.} \] \textit{Coastal Engineering} \textbf{40} (2000)
⇒ equations of motion for the structure:

\[
\sum_{k=1}^{6} \left( M_{jk} + a_{jk} \right) \ddot{x}_k(t) + \int_0^t K_{jk}(t - \tau) \dot{x}_k(\tau) d\tau + C_{jk} x_k(t) = F_{jD}(t) + F_{jnl}(t);
\]

\[ j = 1, 2, \ldots, 6 \]

\[ M_{jk}, \ C_{jk} : \ 1^{\text{st}} \text{ order inertia and hydrostatics} \]
\[ K_{jk}, \ a_{jk} : \ 1^{\text{st}} \text{ order wave radiation impulse response function} \]
\[ F_{jD} : \ 1^{\text{st}} \text{ order wave diffraction exciting force} \]
\[ F_{jnl} : \ \text{Nonlinear forcing. Mooring lines, fender friction, viscous effects, ...} \]
Linear wave-structure interaction with OceanWave3D

At any given time step, the 3D solution is obtained from:

\[ \zeta_t = -\nabla \zeta \cdot \nabla \tilde{\phi} + \tilde{w}(1 + \nabla \zeta \cdot \nabla \zeta) \]

\[ \tilde{\phi}_t = -g \zeta - \frac{1}{2} \nabla \tilde{\phi} \cdot \nabla \tilde{\phi} + \frac{1}{2} \tilde{w}^2 (1 + \nabla \zeta \cdot \nabla \zeta) \]
Multi-block boundary-fitted discretization of the Laplace problem
Each block is mapped to the unit-spaced rectangle
Each block is mapped to the unit-spaced rectangle

- Physical and computational grids defined by the point mappings:
  \[ [\xi(x, y), \eta(x, y)], \text{ and } [x(\xi, \eta), y(\xi, \eta)] \]

- Partial derivatives in the two domains are related by:
  \[
  \partial_x = \xi_x \partial_\xi + \eta_x \partial_\eta, \quad \partial_y = \xi_y \partial_\xi + \eta_y \partial_\eta, \quad \text{etc.}
  \]

- Express the grid transformations in computational space via
  \[
  \begin{bmatrix}
  \xi_x & \eta_x \\
  \xi_y & \eta_y
  \end{bmatrix}
  \begin{bmatrix}
  x_\xi & y_\xi \\
  x_\eta & y_\eta
  \end{bmatrix}
  =
  \begin{bmatrix}
  1 & 0 \\
  0 & 1
  \end{bmatrix},
  \quad \Rightarrow
  \begin{bmatrix}
  \xi_x & \eta_x \\
  \xi_y & \eta_y
  \end{bmatrix}
  =
  \begin{bmatrix}
  x_\xi & y_\xi \\
  x_\eta & y_\eta
  \end{bmatrix}^{-1}
  \]

- Derivatives via operations entirely in the computational space:
  \[
  \partial_x = \frac{1}{J} (y_\eta \partial_\xi - y_\xi \partial_\eta), \quad J = (x_\xi y_\eta - x_\eta y_\xi), \quad \text{etc.}
  \]
A heaving horizontal axis cylinder (linear)

Pseudo-impulsive motion (Gaussian in time)
Fourier transform gives the added mass and damping

\[
\frac{F}{\rho \pi R^2 \omega^2} = \frac{A}{kR}
\]

\[
\frac{B}{\omega}
\]

A Exact

B/ω Exact
Apply the OceanWave3D strategy to the fully nonlinear problem
Fully nonlinear analysis - CFD
Preliminary Work Plan

1. The incident waves
   ▶ Choose a set of test case conditions for the near-shore geometry/bathymetry and the offshore wave conditions
   ▶ Run OceanWave3D for these conditions, store data

2. Mildly nonlinear analysis
   ▶ Complete the OceanWave3D linear hydrodynamics solver
   ▶ Compute IRF’s for a test concept WEC
   ▶ Use WAMSIM to optimize the concept

3. Fully nonlinear analysis
   ▶ Build the nonlinear inviscid solver
   ▶ Apply it to the final concept design for extreme loading and performance analysis