Schematic of the hydrodynamic problem

Deep water waves

Wave forcing at the device

Refraction from the bottom

Triad interaction

Wave farm

Diffraction from coastal boundaries
WP1 Hydrodynamics - Overview
The Standard Wave-To-Wire-Model
Newton’s 2nd law

\[ M_{jk} \ddot{x}_k(t) = f_j^{HD} + f_j^{ML} + f_j^{PTO}, \quad j = 1, 2, \ldots, 6. \]
1. The *Incident Wave* climate at the device location (10 - 50m water depth)
   - Spectral description of long-term wave conditions (DHI)
   - Deterministic modelling of extreme conditions (DTU)

2. Mildly-nonlinear wave-structure interaction - Optimization and long-term loading
   - Linear hydrodynamic coefficients
   - Nonlinear hydrostatics, mooring system, and PTO forces included in the equations of motion (DHI/DTU)
   - Model under development to compute linear hydrodynamics on a variable depth, possibly semi-enclosed domain (DTU)

3. Strongly nonlinear wave-structure interaction - Final design analysis
   - Fully nonlinear inviscid flow solver (PhD - DTU)
   - Viscous flow solver (DHI)
WP-1 The Hydrodynamic Forces

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Consider two coordinate systems:

\[ \mathbf{x} = [x, y, z] \] Fixed in space

\[ \mathbf{\hat{x}} = [\hat{x}, \hat{y}, \hat{z}] \] Fixed to the structure

The frames are related by:

\[ \mathbf{x}_t = [x_1, x_2, x_3] \] A position vector, and

\[ \mathbf{x}_r = [x_4, x_5, x_6] \] A rotation vector
A general vector in the two frames is related by

\[ \mathbf{\hat{x}} = L(\mathbf{x} - \mathbf{x}_t) \quad \mathbf{x} = \mathbf{x}_t + L^T \mathbf{\hat{x}} \]

where

\[
L = \begin{pmatrix}
    c_6 c_5 & s_6 c_5 & -s_5 \\
    -s_6 c_4 + c_6 s_4 s_5 & c_6 c_4 + s_4 s_5 s_6 & s_4 c_5 \\
    s_6 s_4 + c_6 c_4 s_5 & -c_6 s_4 + s_6 s_5 c_4 & c_5 c_4
\end{pmatrix}
\]

and the sequence of rotations is: yaw \((x_6)\), pitch \((x_5)\), roll \((x_4)\); with \(c_i = \cos(x_i)\) and \(s_i = \sin(x_i)\).

The angular velocity of the WEC in the body-fixed frame is:

\[
\mathbf{\hat{\omega}} = [-\dot{x}_6 s_5 + \dot{x}_4, \dot{x}_6 c_4 c_5 + \dot{x}_5 c_4, \dot{x}_6 c_4 c_5 + \dot{x}_5 s_4]
\]

which is related to the time rate of change of the Euler angles by

\[
\mathbf{\dot{x}_r} = \mathbf{W} \mathbf{\hat{\omega}} = \begin{bmatrix}
    1 & s_4 s_5 / c_5 & c_4 s_5 / c_5 \\
    0 & c_4 & -s_4 \\
    0 & s_4 / c_5 & c_4 / c_5
\end{bmatrix} \mathbf{\hat{\omega}}.
\]
Newton’s law at the body Center of Gravity (CG)

\[ E \left[ \begin{array}{c} \ddot{x}_t \\ \dot{\omega} \end{array} \right] = q \]

where

\[ E = \begin{bmatrix} m & -[r_g \times m] L^T \\ (r_g \times m)L^T \hat{i}_g & [r_g \times (r_g \times m)]L^T \end{bmatrix} \]

and

\[ q = \begin{bmatrix} F - m[\hat{\omega} \times (\hat{\omega} \times r_g)] \\ M - m\{r_g \times [\hat{\omega} \times (\hat{\omega} \times r_g)]\} - L^T(\hat{\omega} \times \hat{i}_g \hat{\omega}) \end{bmatrix} \]

with \( F \) and \( M \) the total applied external force and moment (hydrodynamic, hydrostatic, mooring system, PTO...).
The Exact Hydrostatic Forcing

\[ F_s + w = \rho g \int \int_{S_b(t)} z \hat{n} \, dS - mgk \]

\[ M_s = (r_g \times w) + (r_b \times F_s) \]

with \( \hat{n} \) the normal vector to the body surface and \( k \) the unit vector in the vertical.
Test Case: A Heaving Sphere

The grids for the static (left) and dynamic (right) calculations.
Preliminary Results

Heave response to increasingly larger waves:

- Linear
- $H/L=0.020475$
- $H/L=0.051187$
- $H/L=0.07678$
The Next Steps

In Progress:
- Validate the hydrostatic forces using a forced motion
- “Publish” to WP4
- Compare with experimental measurements for the WaveStar Floater

Longer Term Developments:
- Use the waterline defined by the incident wave rather than the still water plane
- Split the diffraction forcing into incident (Froude-Krilov) and scattered components and compute exact Froude-Krilov forces.